ME 443 → Systems & Measurements (Summer 2013)

Section A MW 8:20 – 10:40
F 9:00 – 11:30

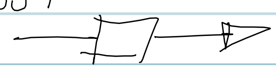
Monday May 13th, 2013

Characteristics of measuring device

static vs dynamic

↓ ↓

input output sensor response



continuous (device) to → discrete (computer)

NOTE:
Important to communicate with "confidence intervals."

Here we will use a systems approach
(input → system → output)

NOTE: Fourier → tells us which frequencies are in a signal

Static Calibration:

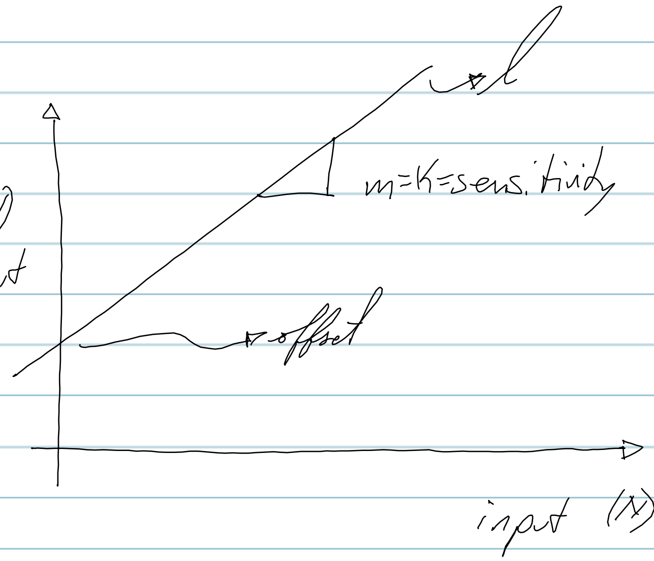
Marrage: what is desired output?

→ you want to know the relationship between input and output.

↳ Done by putting in "Known Values"

↳ Done in/for increasing and decreasing trends ex (Force Measurements)

input (N)	output (V)
0	0.9
10	1.0
20	2.1
⋮	
60	4.2

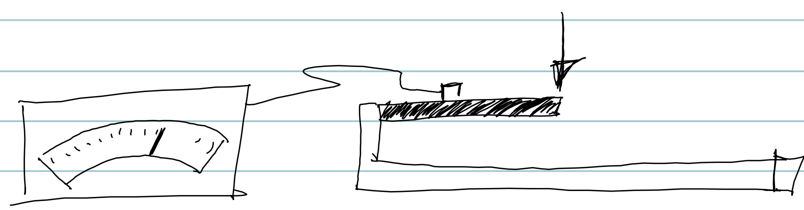


Systems: input → [measuring Device] → output

"truth" "measured"

→ there will be a difference

FORCE



FORCE → [strain $\left(\frac{in}{in}\right)$ → gauge $\left(\frac{\Delta R}{R}\right)$ → Wheatstone (V) Bridge (signal conditioning) → Analog→Digital converter →] estimated Voltage

System: [Sensing element] — [Signal Conditioning] — [Data Aquisition]

CBFD → cantilever beam force transducer

Calibration

$$\text{output} = K \times \text{input} + \text{offset}$$

$$O = K \cdot I + b$$

$$I = \frac{O - b}{K}$$

True only in usable range:

input range: 0 to 60N

input span: (max − min)

output range: O_{min} to O_{max}

output span: $O_{max} - O_{min}$

"full scale deflection"

Linear Range: ex (10N to 40N)
Usable " "

max non-linearity: maximum difference between a data point & a line
of data set (not device)

as % fsd

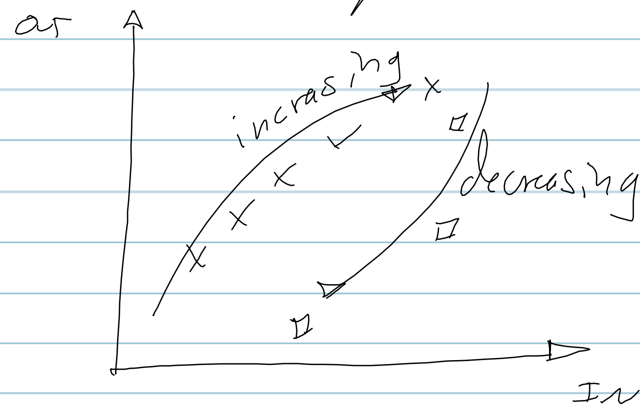
$\frac{1.8}{3.3} = 0.54$ FSD

should be within 5% for purchased measuring devices

Process Specific:

Hysteresis (sensitivity to direction) (usually undesirable)

↳ a dependence on direction



$$|\text{Hysteresis}| : |O_{inc} - O_{DEC}|$$

$$\text{Max HYS } (\% F_{SD}) : \frac{|O_{inc} - O_{DEC}|_{max}}{F_{SD}}$$

Input Resolution: $\Delta I = \frac{\Delta O}{K}$

Output Resolution:

how much can input change before output changes?

OR the largest change in input which does not necessitate an output

We want ΔI to be small (high resolution)

1. $O = KI + b$
2. Usable Range

} Assuming input is Knowable

3. ΔI

(Repetition may allow for randomness from noise)

↳ calibration may be required before & after each use (sometimes during)

Increasing Accuracy of "Known" inputs:

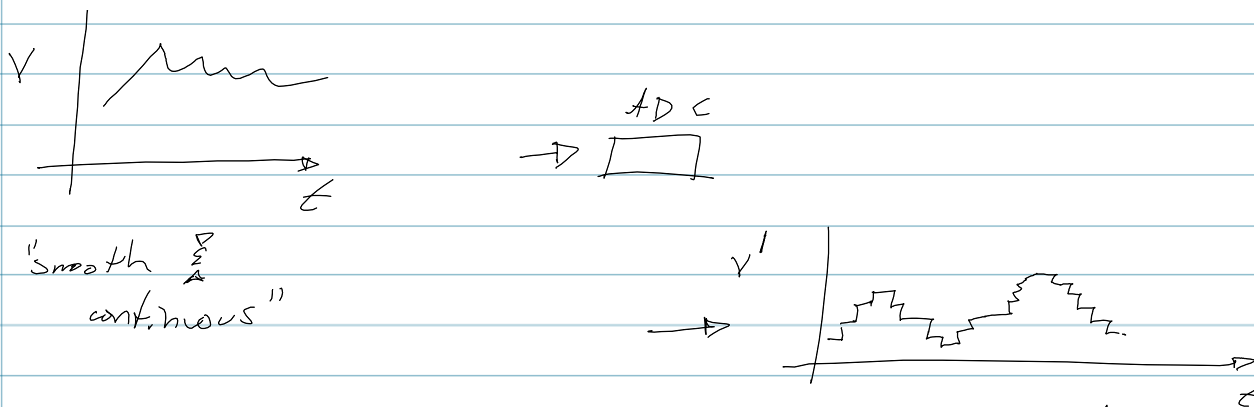
- Instrument to be calibrated (thermocouple)
- Lab standard (thermometer)
- Transfer standard (From thermometer Manuf)
- Ultimate N.I.S.T.

WEDNESDAY May 15th, 2013

DATA Aquisition → Analog to Digital conversion

→ once you infer reading you've digitized it

analog signal → [ADC] → 0011101, 100110, 1001111 ↓ time



"smooth & continuous"

"discrete & discontinuous"

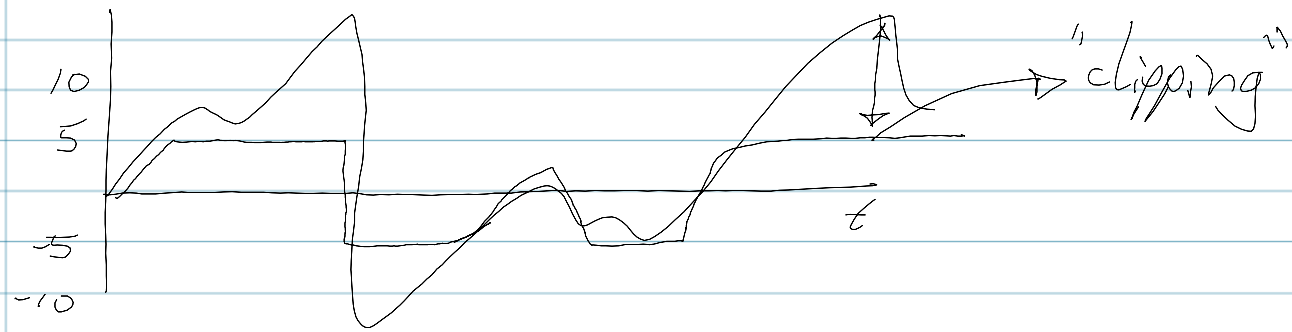
you would like to make ADC choices so input and output look alike.

"ADC" Parameters:

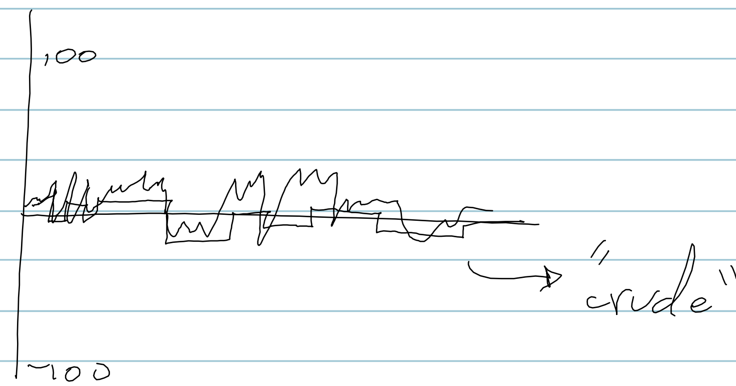
(fixed for hardware)

1. input range
2. # of bits
3. Sample rate

1) Input Range → Range of voltages ADC can see



OR oppositely



Balance: clipping vs. crude approx.

Set: $|V_{max}| = \text{Amplitude (of input)} \times 120\%$

↳ which will allow for small fluxuations

↳ start w/ max input and adjust accordingly

Quanitization:

"input range"

$$Q = \frac{V_{max} - V_{min}}{2^n - 1} = \frac{IR}{2^n}$$

3 bits	2 bits
111	11
110	00
101	10
100	01
010	
011	
001	
000	

$\Big\}$ # of steps 2^n

"nominal": I.R.

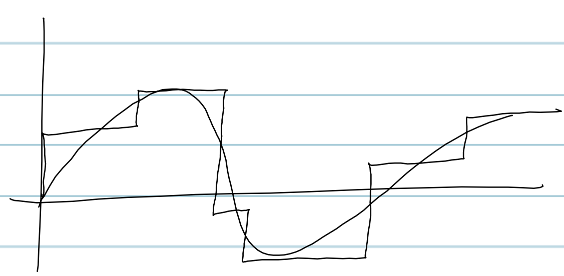
"True": $-R$ to $(R-Q)$

NOTE: we would like Q to be small

Bits → too big: cost \$; storage; processing

→ too small: Q is big & resolution is poor

for Q to be small; # of bits must be large



Q is too large: "crude"

converting from voltage to code

$$C = NIT\left(\frac{V_i - V_{ADC\,min}}{Q}\right)$$

NIT ↑ nearest integer to...

⟹

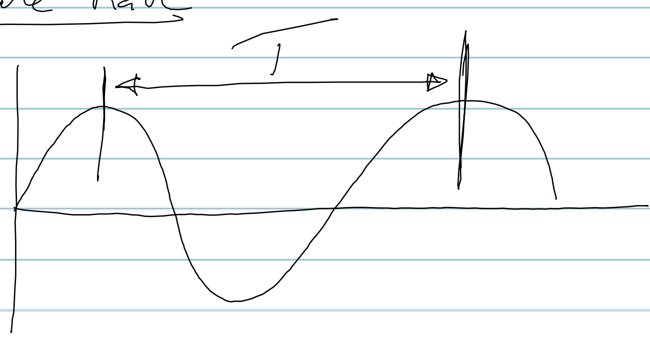
$\approx V_i = cQ + V_{ADCmin}$

(approximation, just for understanding)

$$\boxed{\hat{V}_i = cQ + V_{ADCmin}}$$

MAX ERROR: $\frac{Q}{2}$ (quanitization error)

3) Sample Rate:



$$f_{sig} = \frac{1}{T}$$

If $f_{sig} < 2 f_{samp}$

↳ phenomenon called aliasing

$\therefore$ Nyquist Criterion:

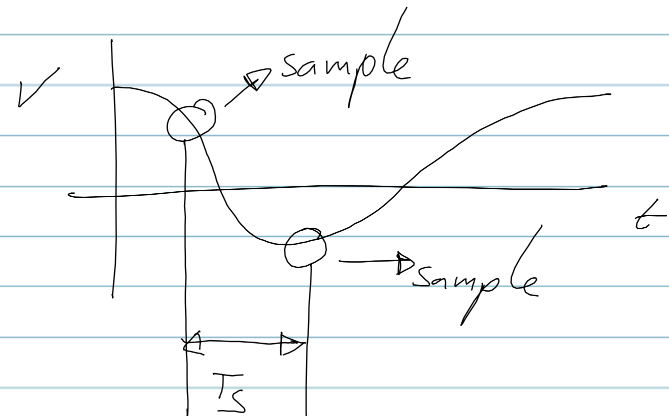
$$f_{samp} \underset{\text{must be}}{\geq} 2f_{sig}$$

$$V(t) = A\sin(2\pi f_{sig}\, t)$$

$$V(t) = A\sin(2\pi f_{sig}\, n\, T_s)$$

time between samples

increments



Derivation:

$$A\sin(2\pi f_{sig} n T_s) = A\sin(\text{'' ''} + 2\pi n K)$$

$$\vdots$$

$$= A\sin(2\pi n T_s (f_{sig} + K f_{samp}))$$

$\Rightarrow$

A signal with this frequency:

$$f_{sig} + K f_{samp}$$

will appear as:

$$2\pi f_{sig} n T_s$$

(In practice $f_{samp} \approx 10 f_{sig}$)

44.1 KHz

→ humans hear @ (0) to 20 Hz

If f_{samp} is fixed

→ [filter] → ADC [$f_{samp} = 1\,Hz$] → Data

→ filter everything above 500 Hz

↓

"low pass" filter

(anti-aliasing)

FRIDAY May 17, 2013

Aliasing

$$f_{TRUE} = K f_s \pm f_{appear}$$

$$\boxed{f_{sig} = K f_{samp} \pm f_{appear}}$$

for example:

let $x(t) = 15 \sin(200\pi t)$

$f_s = 400, 100, 40 \text{ Hz}$

$f = 200\pi \frac{rad}{s}$
$= 100 \text{ Hz}$

$\omega = 2\pi f$

$\uparrow \frac{rad}{s}$ $\nwarrow \frac{cyc}{sec}$ (Hz)

case: $f_{samp} = 400\text{Hz}$ $100 \text{ (Hz)} = K(400\text{Hz}) \pm f_{apparent}$

for $K=0 \Rightarrow 100\text{Hz} = f_{apparent}$ $(\pm)$ $\therefore f_{app} = 100 \text{ Hz}$

Check Nyquist criteria

is $f_{app} < \frac{f_{samp}}{2}$?

$100 < \frac{400}{2}$ ✓ check ✓

because

$$\boxed{f_{app} = 100 \text{ Hz}}$$

case $f_{samp} = 100\text{Hz}$ $\Rightarrow$ $100 \text{ (Hz)} = K(~~4~~100 \text{ Hz}) \pm f_{apparent}$

for $K = 0 \Rightarrow f_{app} = 100 \text{ Hz}$ check

is $100 < \frac{100}{2}$ NO

$K = 1$ ...

$$\boxed{f_{app} = 0 \text{ Hz}}$$

example: (continued)

$f_{samp} = 40\,Hz$

$100\,Hz = 40\,Hz \cdot K \pm f_{app}$

for $K = 0 \Rightarrow f_{app} = 100\,Hz$ No

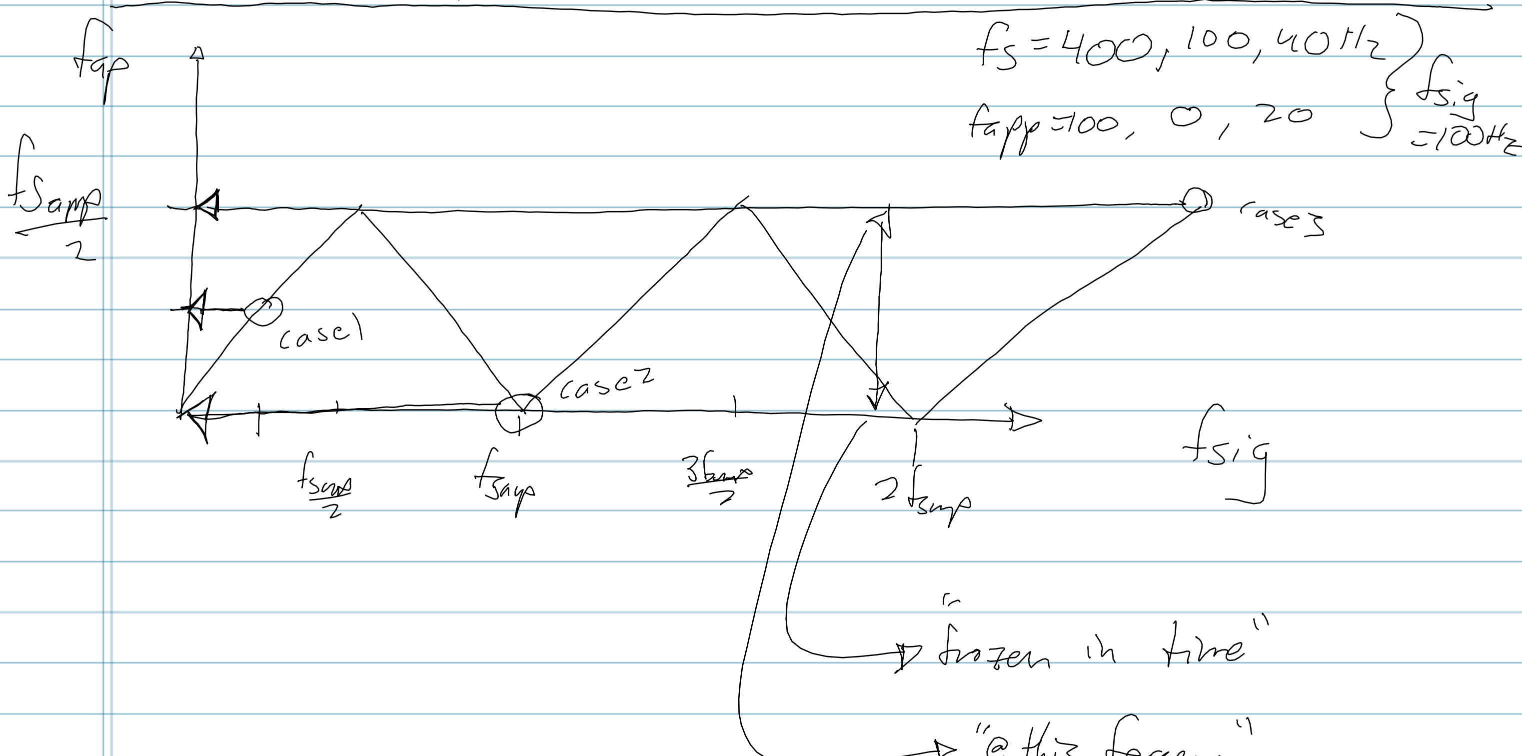
Now, $K = 1$

$100\,Hz = 40\,Hz \cdot 1 \pm f_{app} \Rightarrow f_{app} = 60\,Hz$

is $60 < \frac{40}{2}$ No

Now, $K = 2 \ldots$ is $20 \leq \frac{40}{2}$ Yes

$\boxed{f_{app} = 20\,Hz}$



ADC →

1. Input Range
2. Sampling
3. Resolution

} parameters

By looking @ FFT, we can play w/ "harmonics" (different frequencies) at different amplitudes (this can mimik the difference between different musical instruments)

→ Aliasing discussion, named appropriately

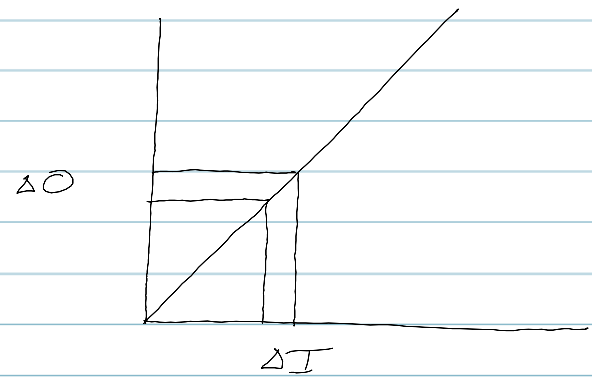
→ Water Aliasing → Speaker w/ tube fountain

Camera → 24 fps

speaker 24 Hz

→ might not work with human vision because of "blurring" static calibration

Homework question:



$\Delta I = \frac{\Delta O}{K}$

$\Delta O = Q$ "delta "O" is "Q""

Monday May 20th, 2013

Statistics - "we want to quantify confidence in data"

Questions 1. R.B. experiment

2. temp only in time (1 samp) conf. intv.

confidence intervals:

→ Probability dist. function of the means

↳ we want to know the statistical qualities of the Means of the data set

Theory: if data is Gaussian & N is large then the distributions of the means will be gaussian

NOTE: (1 standard deviation is 68%)

↳ this applies to data points & to means

We can now find a range bounding the true mean. Then we can say that we are 68% confident that the true mean falls within 1 std. dev. of measured/or calculated mean "spread of means will be 'tighter' than spread of data"

GOAL: Report unknowable values with a quantitative level of confidence

example $N=100$ given: $\hat{\mu}_x = 70.5°F$

$\hat{\sigma}_x = 3.1°F$

find: 95% C.I. for true mean

$95\%\ CI = \bar{X} \pm Z_Q\ \sigma_{\bar{X}}$ — this is "true" std. dev.

$70.5 \qquad (1.96)\left(\frac{3.1}{\sqrt{100}}\right)$

from Table

$69.89 < \text{True Mean} < 71.1$

True mean $\stackrel{?}{=}$ the mean of the means.

99% C.I.

$CI = \bar{X} \pm 2.5\ \sigma_{\bar{X}}$

69.69°F to 71.3°F

As our ability to collect data points decreases, our data will follow a less and less recognizable Gaussian curve, we move to the "student-t" distribution

→ NOTE: @ a given confidence; the range will expand, as n decreases.

(a lot
(for two factors))

$\nu = N - 1$ → "loss of a degree of freedom due to an assumption..."

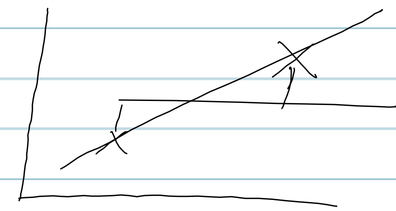
PROPAGATION OF ERROR

$u = f(x, y, \ldots)$

⋮

Fitting Functions to Data:

"sum of least squares"



↳ why we use line with least "error"

L.S.F. (least square fitting)

x^2	x	y	xy
⋮	⋮	⋮	⋮
$\sum x^2$	$\sum x$	$\sum y$	$\sum xy$

$\left.\begin{array}{l}\sum x \\ \sum y\end{array}\right\}$ use in (a_0, a_1) expressions

$\sum xy$

R^2 → case by case basis for "fitting quality"

↳ is a measure of how well your trend fits your data

NOT how well your trend models the system!!

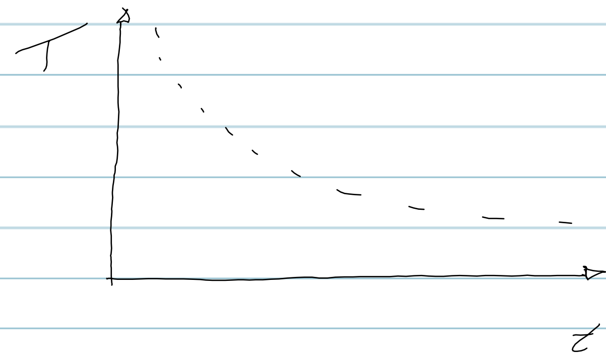
(of course)

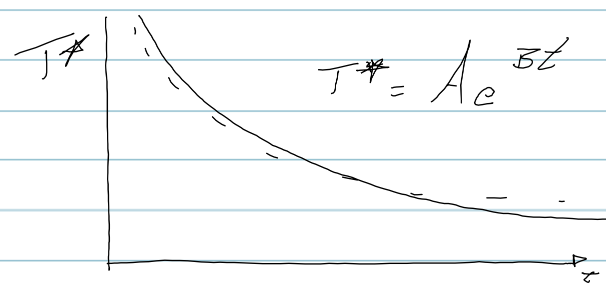
WEDNESDAY May 22nd, 2013

Test on
1. S. Calibration
2. A to D C
3. Statistics

$$T^* = \frac{T - T_\infty}{T_i - T_\infty} = e^{\left[-\left(\frac{hA_s}{\rho V c}\right)t\right]}$$

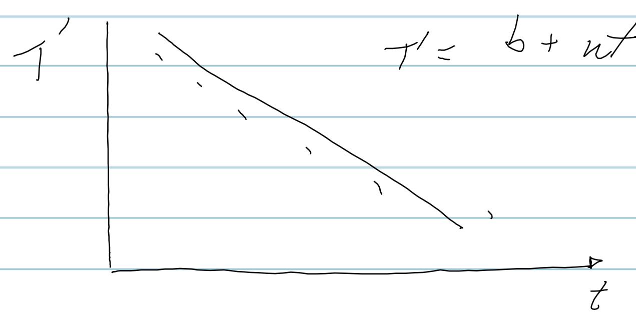
time	Temp	T^*
⋮	⋮	
⋮	⋮	





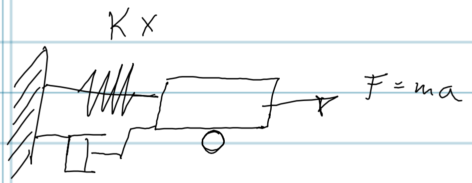
NOTE: You may also Natural log "T^*" $(\ln(T^*))$

$$T' = \ln(T^*) = -\left(\frac{hA_s}{\rho V c}\right)\cdot t$$



Differential Equations:

(How all systems behave in time)

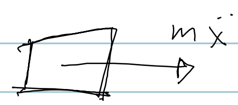


$c\dot{x}$

$\xrightarrow{+} \Sigma F$

$$-Kx - c\dot{x} + F = m\ddot{x}$$

KD



$$m\ddot{x} + c\dot{x} + Kx = F(t)$$

$\downarrow$

2nd Order D.E.
linear
inhomogeneous

transient & steady-state options
(homogenous) (particular)

Add them together:

$$X_{TOT} = X(t) + X_{SS}(t)$$

1. Transient:

$$x = Ae^{\lambda t}$$
$$\dot{x} =$$
$$\ddot{x}$$

Plug into D.E.

↳ "Characteristic equation"

2. Steady-state

Let $X_{SS}(t)$ = form of input

↳ (assumption)

then do as before: differentiate and plug into D.E.

then solve for constants or coefficients

Now:

$$X_{TOT} = \underset{trans}{X(t)} + \underset{steady\text{-}state}{X(t)}$$

still has 1 unknown; we use initial conditions to get exact solution

First Order: τ: time constant; K: sensitivity

2nd Order: $\omega_n \rightarrow$ Natural frequency

$\xi \rightarrow$ damping ratio

$K \rightarrow$ static sensitivity

Keep in mind "method of undefined coefficients"

$\rightarrow$ where any $A\sin$ or $B\cos$ can be modeled with "$= A_1\cos + A_2\sin$"

$Ae^{\gamma t}$ $\rightarrow$ integrating factor

Headed towards: dynamic ability to measure

$\rightarrow$ what's good for measuring

Statistics h.w. due Friday

$\downarrow$

Exam $\rightarrow$ Monday (no class)

$\downarrow$

Wed Today's h.w. due (D.E. stuff)

Exam is open notes $\rightarrow$ Review homework

WEDNESDAY May 29th, 2013

Numerical Integration

$a\ddot{x} + b\dot{x} + cx = f(t)$ (2nd Order O.D.E.)

$x_1 = x$

$\dot{x}_1 = x_2$

$x_2 = \dot{x}$

computers love to solve 1st order O.D.E.'s

$\dot{x}_2 = \ddot{x} = \frac{1}{a}\left(f(t) - b\dot{x} - cx\right)$

So,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{f(t) - bx_2 - Cx_1}{a} \end{cases}$$

$dx/dt = x_2$ → approximation, not unlike ADC → "RUNGE KOTTA"

We can only talk about "linear" differential equations.

All physical systems are inherently non-linear.

Matlab → clf

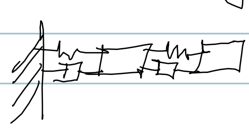
$t_0 = 0;\ tf = 50{,}000;$

$X_0 = [0;1]$

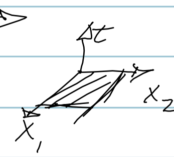
[t, x] = ode45('sdof', [to, tf], X0);

subroutine

Discussion on Degrees of freedom



State-Space Plot



Frequency Response Functions

1st ORDER

$$\tau \dot{y} + y = K\, x(t)$$

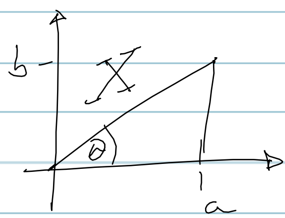
output input

To GET FRF:

magnitude & phase

frequency ω

1) assume input to be $x(t) = \underbrace{\underline{X}\, e^{j\omega t}}_{\text{generic sinusoid}}$



$$|X| = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

2) Then $y(t) = \underline{Y}\, e^{j\omega t}$

$$\tau j\omega Y e^{j\omega t} + Y e^{j\omega t} = K \underline{X} e^{j\omega t}$$

$$Y(1 + \tau j\omega) = K \underline{X}$$

OR

$$\boxed{\frac{\text{out}}{\text{in}} \quad \frac{Y}{X} = \frac{K}{1 + \tau j\omega} = T(j\omega)}$$

$$\text{if } T = \frac{a + jb}{c + jd}$$

$$|T| = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

$$\angle t = \angle num - \angle den = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(d/c\right)$$

ω	$\lvert T \rvert$	$\angle$ (deg)
0	1	0
5	0.89	-26.6
10	0.71	-45
15	0.55	-56
20	0.44	-63

→ means a lagging behind

Electrical or Thermal systems can be 1^{ST} order

But Mechanical systems are 2^{nd} order

@ high frequencies, (same input magnitude)

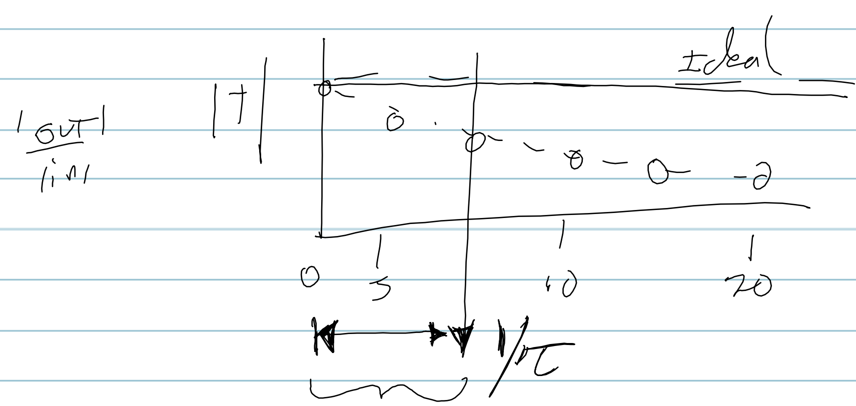
→ for 1^{st} order → displacement becomes zero

$$\frac{Y}{X} = \frac{K}{1+\tau j\omega}$$

$\omega = 0$ this is static calibration curve

Output Voltage

may chang as the (ω) of input increses.



Usable range of 1^{ST} order system

If $x(t) = A \sin \omega_1 t$

$$\text{Then} \quad y(t) = \underbrace{|T| A}_{\text{New Magnitude}} \sin\left(\underbrace{\omega t + \phi}_{\text{New Phase}}\right)$$

ex

$x(t) = 10\sin(5t)$

$y(t) = |T|\, 10\sin(5t + \angle T)$

$y(t) = (0.89) \cdot 10 \cdot \sin(5t + (-26.6))$

ex

[illegible]

$x(t) = 1\cos(17t + 40)$

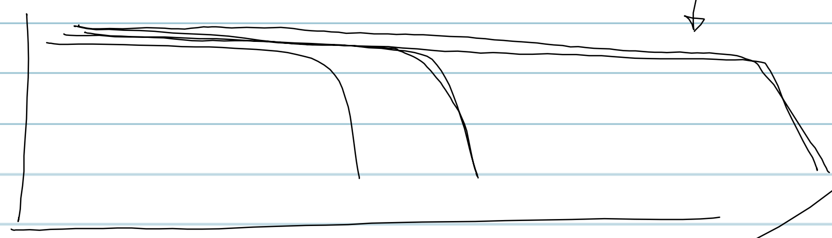
$y(t) = |T_{17}| \cos(17t + 40 + \underbrace{\angle T}_{-60})$

$= 0.5\cos(17t - 20)$

This stuff is the steady-state response of a system.

Static

Dynamic → usable range (of frequency inputs) "better"



in terms of dynamic usable range for a measuring system

2nd Order:

$$m\ddot{y} + c\dot{y} + Ky = x(t)$$

$$x(t) = X e^{j\omega t}$$

$$y(t) = Y e^{j\omega t}$$

$$\dot{y} = j\omega Y e^{j\omega t}$$

$$\ddot{y} = -\omega^2 Y e^{j\omega t}$$

Substituting & cancelling

$$\boxed{T = \frac{OUT}{IN} = \frac{1}{(K-m\omega^2) + cj\omega}}$$

$$|T| = \frac{1}{\sqrt{(K-m\omega^2) + (c\omega)^2}}$$

$$\angle T = -\tan^{-1}\left(\frac{c\omega}{K-m\omega^2}\right)$$

MATLAB

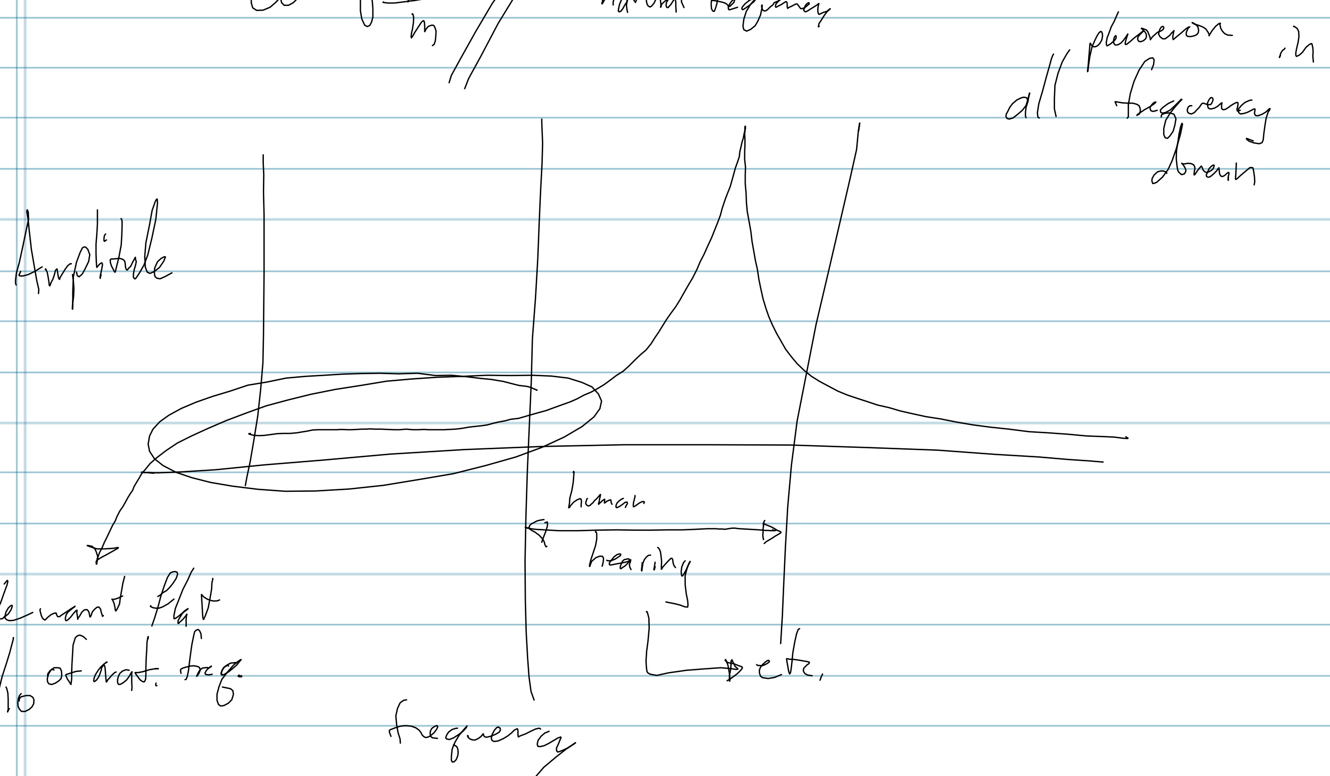
. Dot

$T = K./(1+tau+j*w)$

Term by term (b/c MATLAB handles vectors) ((not numbers))

The system resonates @ "natural fequency"

$\omega = \sqrt{\frac{K}{m}}$ // "natural fequency"



Homework due Friday (from Monday)

assigned Today due Monday

+ (small assignment w/ "practice exam"

Friday 9:00am

FRIDAY May 31st, 2013

Homework #4 (Due Monday)

Second Order

↳ #3 time sol $x(t) = \ldots$

#4

↳ change m, c, K (parameters) hit pm

↳ stiffness

↳ numsim.m → calls sdof.m

1st ORDER

parameters

K & τ

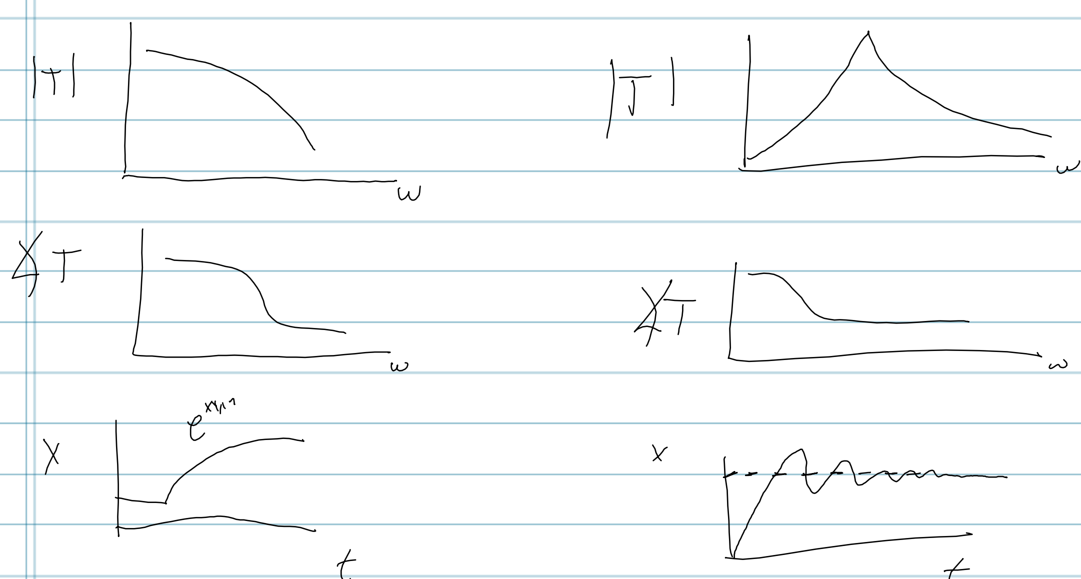
$$T = \frac{K}{1 + \tau j\omega}$$

↳ frequency based

D.E. → time based

You can visualize the behavior of a system in both the time & frequency domains.

1st ORDER 2nd order



Today: System identification (SI)

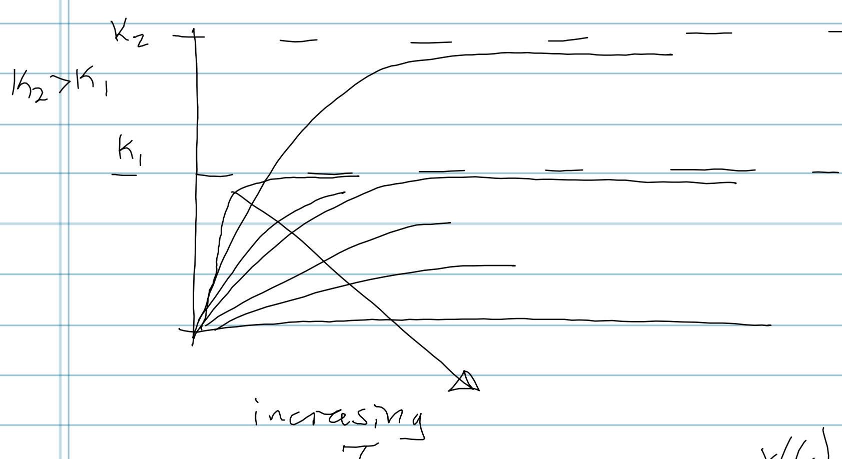
↳ Knowing input & output, so we infer something about the system

1^{st} Order → K, τ

2^{nd} Order → m, c, ω

1^{st} Order (time)

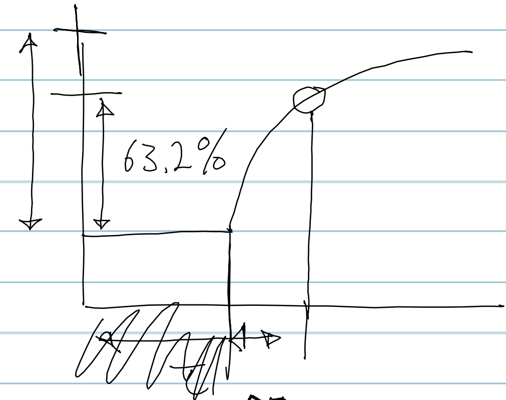
$$K = \frac{\Delta OUT}{\Delta IN} = \frac{X_f - X_0}{10°C \rightarrow \text{input}}$$



$$x(t) = X_0 + (x_f - X_0)\left[1 - e^{-\frac{1}{\tau} \cdot t}\right]$$

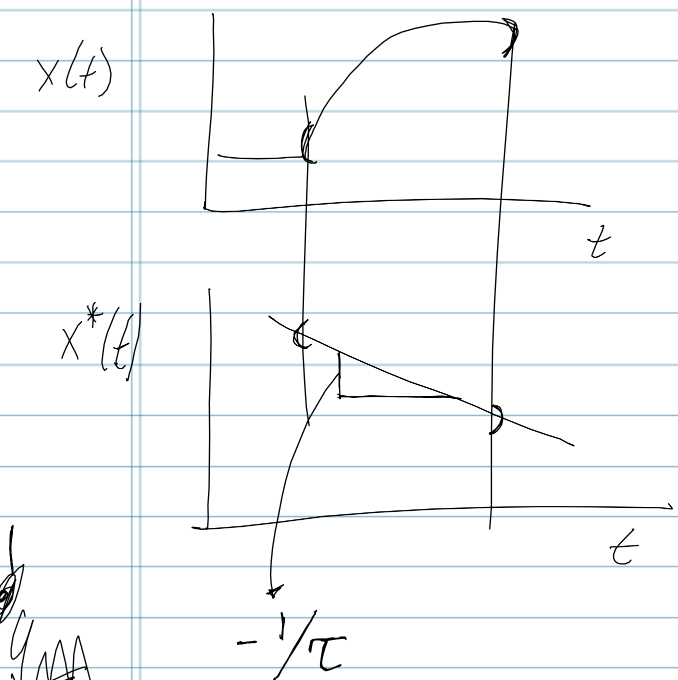
$$\frac{x(t) - X_0}{X_f - X_0} = 1 - e^{-\frac{1}{\tau} \cdot t}$$

$\tau = 1$	$\tau = 2$	$\tau = 3$
0.632	0.865	0.950



"Counting Taus"

Log linearization



$\cancel{x^* = \ln(x)}$

$$x^*(t) = \ln\left[\frac{x(t) - x_f}{x_0 - x_f}\right]$$

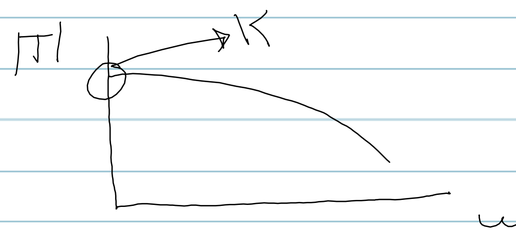
$$= \ln\left(e^{-\frac{1}{\tau}t}\right)$$

$$x^*(t) = \frac{-1}{\tau} \cdot t + 0$$

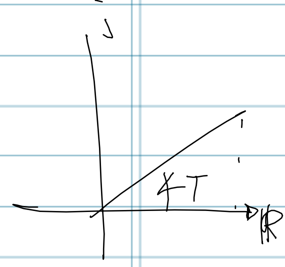
Benifit → entire exponential is used

1st Order (in frequency domain)

frf $\quad T(j\omega) = \dfrac{K}{1 + \tau j\omega}$

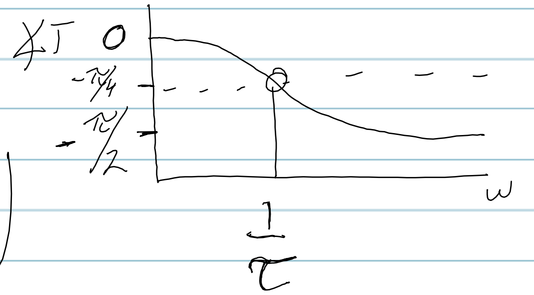


$$|T| = \frac{K}{\sqrt{1^2 + (\tau\omega)^2}}$$

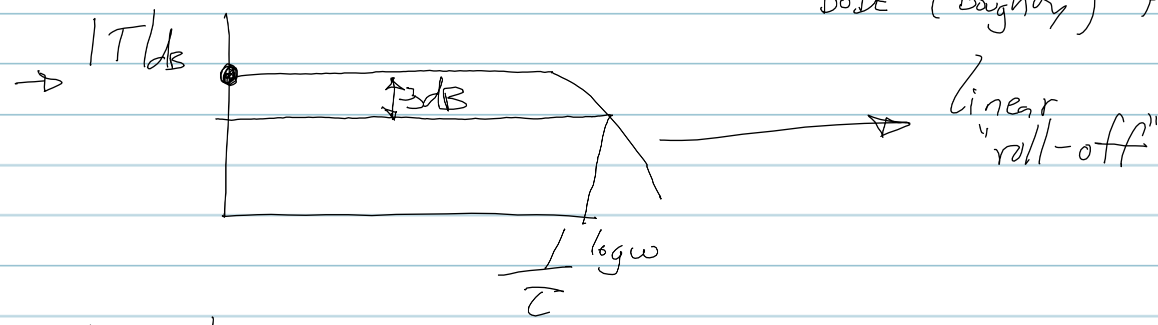


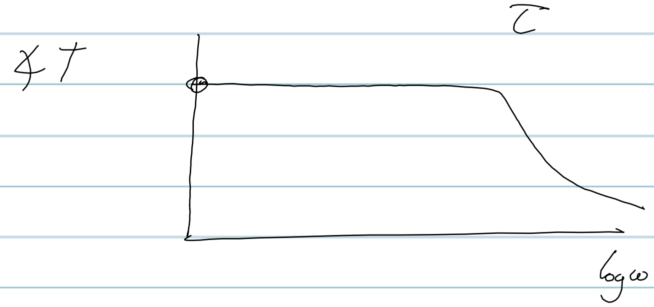
$$\measuredangle T = -\tan^{-1}\left(\frac{\tau\omega}{1}\right)$$

$$= -\tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$$



BODE ("Boughdy") Plots





$$X_{dB} = 20 \log_{10} \left(\frac{X}{X_{ref}} \right)$$

usually 1

2nd Order → Time

Logarithmic Decrement

$y_i \rightarrow$ any peak

$$\delta = \frac{1}{n} \ln\left(\frac{y_i - y_f}{y_{i+n} - y_f}\right) = \zeta \omega_n T_d = \zeta \omega_n \frac{2\pi}{\omega_d}$$

$$= \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

where

$$\zeta = \sqrt{\frac{\delta^2}{\delta^2 + 4\pi^2}}$$

2nd Order (IST)

requires: m, c, k

OR

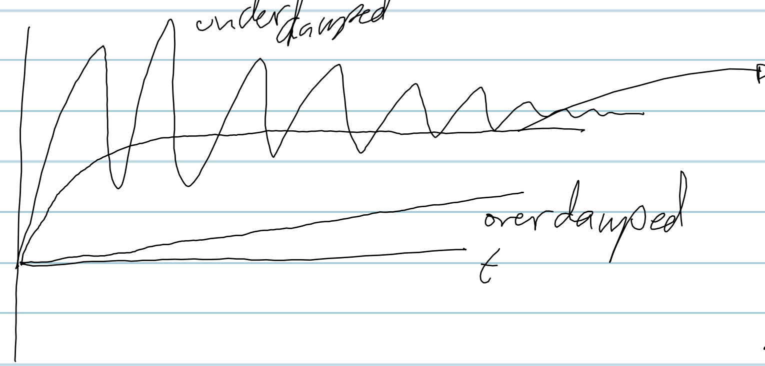
ζ, ω_n, K

$\zeta > 1$ overdamped

$\zeta = 1 \rightarrow$

$\zeta < 1$ underdamped

ζ = damping ratio $\Rightarrow$ ratio of system damping to critical damping c^*



critically damed → no oscilation but quikest

↳ best solution for vibration free

resonant frequency

$$\omega_n = \sqrt{k/m}$$

$$2\zeta\omega_n = c/m$$

$$\underline{K} = \frac{1}{k}$$

about ω_n:

↳ (without damping, not physically possible (purely mathematical)

$$\delta = \ln\left(\frac{y_1 - y_f}{y_2 - y_f}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\zeta = \sqrt{\frac{\delta^2}{\delta^2 + 4\pi^2}}$$

$$T_d = \frac{2\pi}{\omega_D}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_d = \omega_n\left(\sqrt{1-\zeta^2}\right)$$

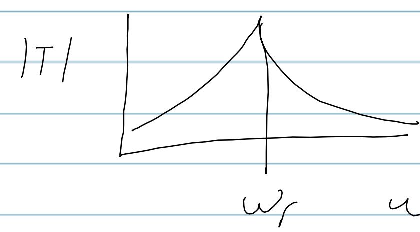
"damped natural frequency"

↳ Frequency of free vibration

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

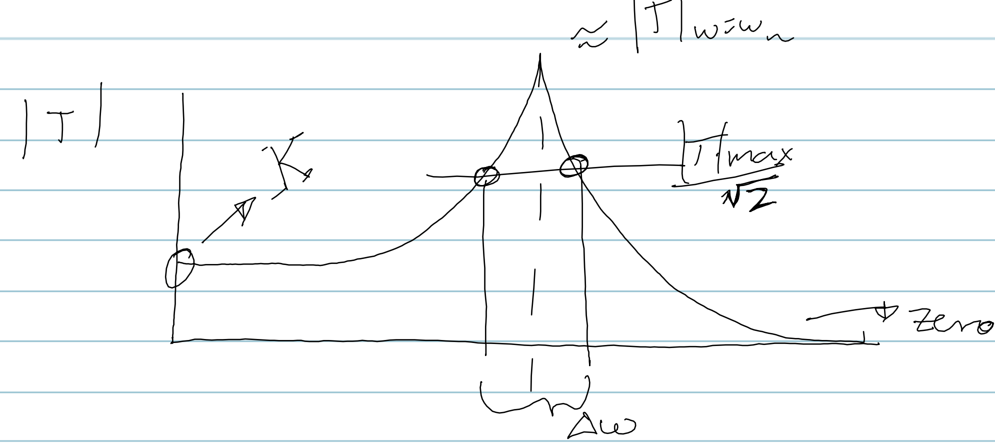
"resonant freq"

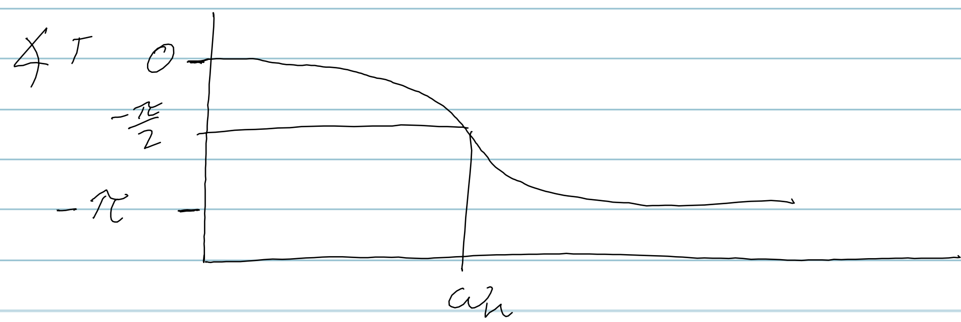
↳ "peak on frf



2^{nd} Order $\rightarrow$ Frequency

$$T = \frac{K}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + 2\zeta j \frac{\omega}{\omega_n}}$$





where $|T| = \dfrac{K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$

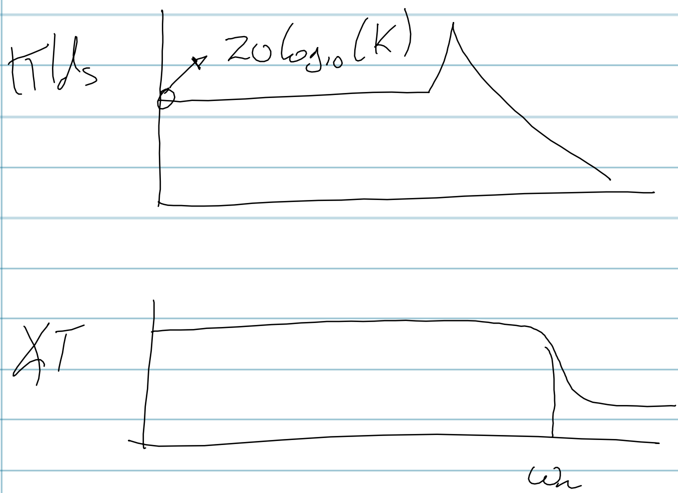
$$\measuredangle T = -\tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$$

Amplification factor

$$\frac{|T|_{\omega=\omega_n}}{|T|_{\omega=0}} = \frac{\frac{K}{2\zeta}}{K} = \frac{1}{2\zeta}$$

OR "Half Power Method"

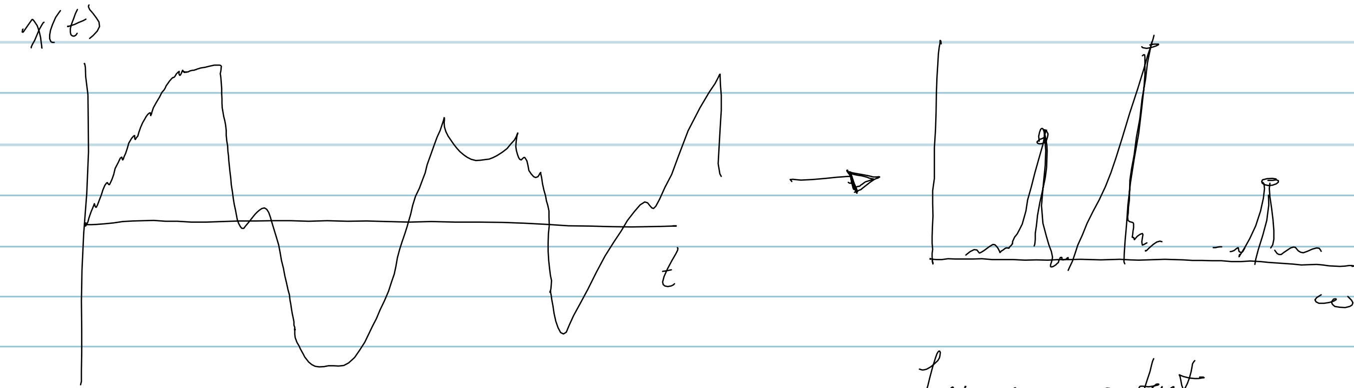
$$\xi = \frac{\Delta \omega}{2\omega_n}$$ → proportional to damping



BODE Plots

Monday June 3rd, 2013

Spectral Analysis - Fourier Series



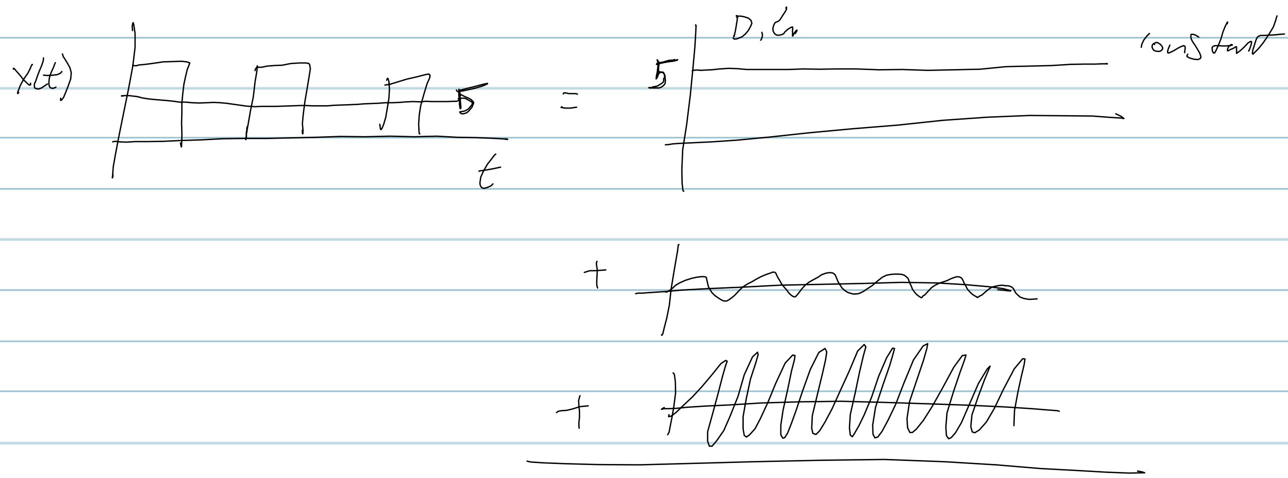
frequency content

$$x(t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_1 t) + B_k \sin(k\omega_1 t)$$

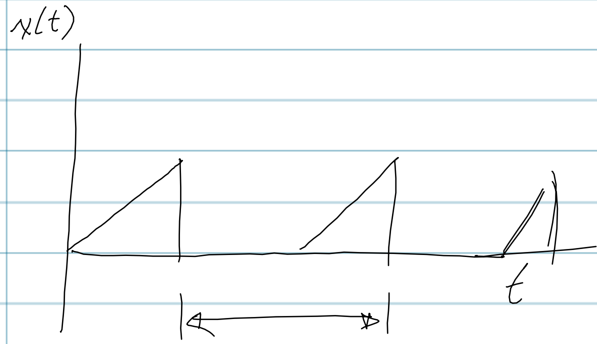
D.C. term OR offset

A.C. terms.

could say sine wave w/ Amplitude = 0



The constant term accounts for "offset", because sine and cosine are centered about zero.



T : period of $x(t)$

$$\omega_1 = \frac{2\pi}{T}$$

To perform Fourier Analysis, we must compute $\frac{A_0}{2}$, A_K, B_K

We start our journey with: (integrating)

$$\int_0^t x(t)\,dt = \int_0^T \frac{A_0}{2}\,dt + \sum_{K=1}^{\infty} \int_0^T A_K \cos(\alpha) + B_K \sin(\alpha)\,dt$$

$$\boxed{\frac{A_0}{2} = \frac{1}{T}\int_0^T x(t)\,dt}$$

Next, for A_K, we multiply each term by $\cos(m\omega_1 t)$ → m is any integer

$$\int_0^T x(t) \cos m\omega_1 t \, d = \int_0^T \frac{A_0}{2} \cos m\omega t + \int_0^T \sum A_K \cos K \cos + \int_0^T \sin\cos$$

unless $K = m$ zero

$$\int A_K \cos^2 K\omega T = A_K \, \tfrac{1}{2}$$

$$\int_0^T x(t) \cos K\omega t \, dt = A_K \frac{T}{2}$$

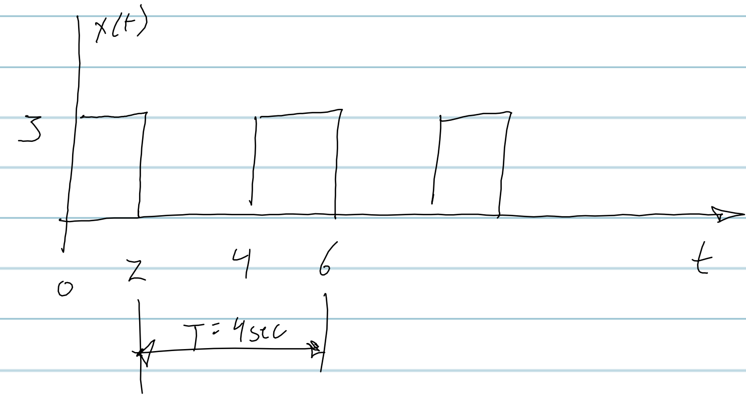
$$\boxed{A_K = \frac{2}{T}\int_0^T x(t) \cos(K\omega t)\, dt}$$

for B_K we multiply by $\sin(m\omega t)$

$$\boxed{B_K = \frac{2}{T}\int_0^T x(t) \sin(K\omega t)\, dt}$$

for Example:

Given: $x(t)$, find: $\frac{A_0}{2}, A_K, B_K$



$$\omega_1 = \frac{2\pi}{4}$$

$$\frac{A_0}{2} = \frac{1}{T}\int_0^T x(t)\, d_T = \frac{1}{4}\left[\int_0^2 3\, dt + \int_2^4 0\, dt\right] = \frac{3}{2}$$

$$A_K = \frac{2}{T}\int_0^T x(t)\cos(K\omega t)\, dt$$

$$= \frac{2}{4}\left[\int_0^2 3\cos(K\omega t)\, dt + \int_2^4 \cos\omega t\right]$$

$u = K\omega t$
$du = K\omega\, dt$

$$= \frac{2}{4}\left(\frac{3}{K\omega}\right)\sin(K\omega t)\Big|_0^2 = \frac{3}{2}\frac{1}{K\omega}\sin(2K\omega)$$

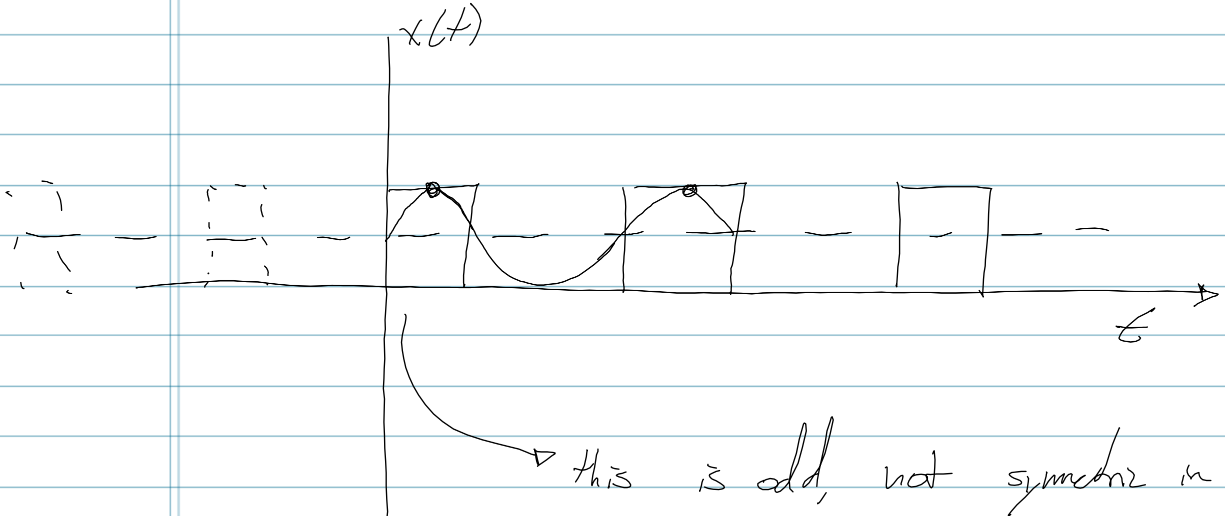
$$= 0$$

$$B_K = \frac{2}{T}\int_0^T x(t)\sin(K\omega t) = \ldots = \frac{3}{K\pi}\left(1-\cos(K\pi)\right)$$

$$x(t) = \frac{3}{2} + \left[\underset{K=1}{1.91\sin\left(\frac{2\pi}{4}t\right)} + \underset{K=2}{0.64\sin\left(\frac{3\cdot 2\pi}{4}t\right)} + \underset{K=\ldots}{\ldots}\right]$$

"Any signal can be approximated by a collection of sines and cosines."

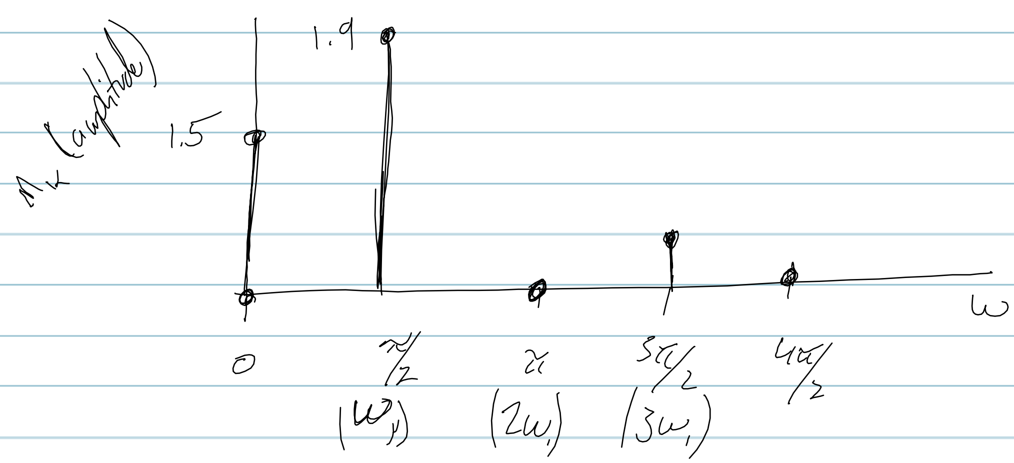
$$x(t) \approx \frac{A_0}{2} + \sum_{K=1}^{N} A_K \cos(K\omega_1 t) + B_K \sin(K\omega_1 t)$$



this is odd, not symmetric in time

NOTICE: all evens (A_K, cosines) are zero since input is odd, we have all odds (B_K, sines)

$M_K = \sqrt{A_K^2 + B_K}$



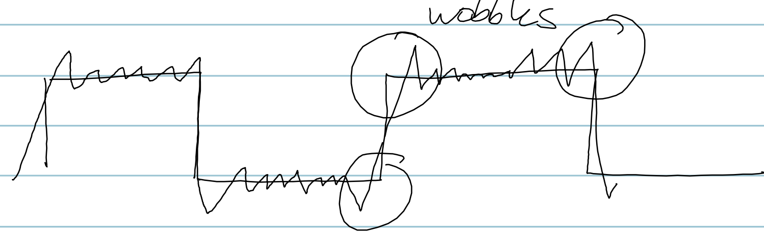
$$x(t) = \sum_{k=1}^{\infty} M_k \cos(k\omega_1 t - \phi_k)$$

$$\phi_k = \tan^{-1}\left(\frac{B_k}{A_k}\right)$$

We now look at a "pulse plot"

or a "pulse wave"

Gibbs phenomenon → Fourier's approximation has trouble around points of discontinuities, "wobbles"



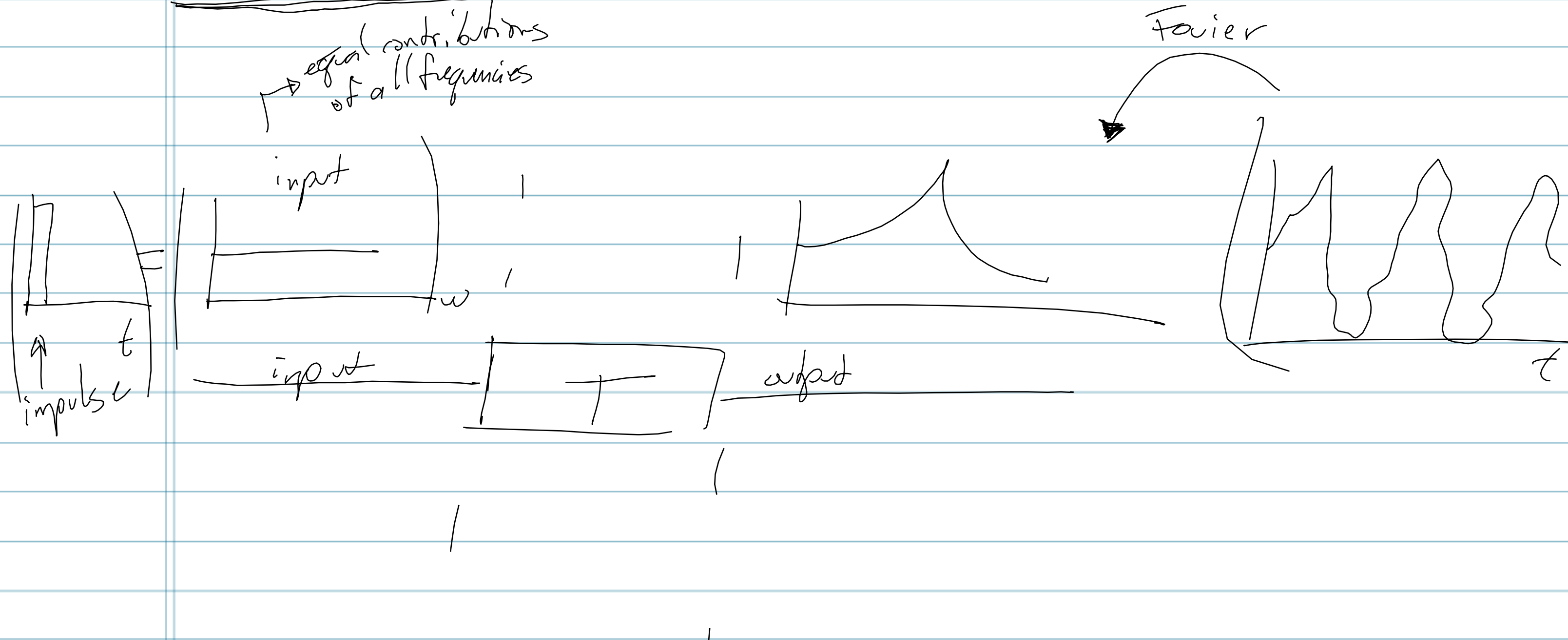
etc.

An "impulse" contains all frequencies in input, so all are passed to output

↳ resonant frequencies will be pronounced!

Bell ringing!?!?

Now we can use *any* input in our systems perspective.



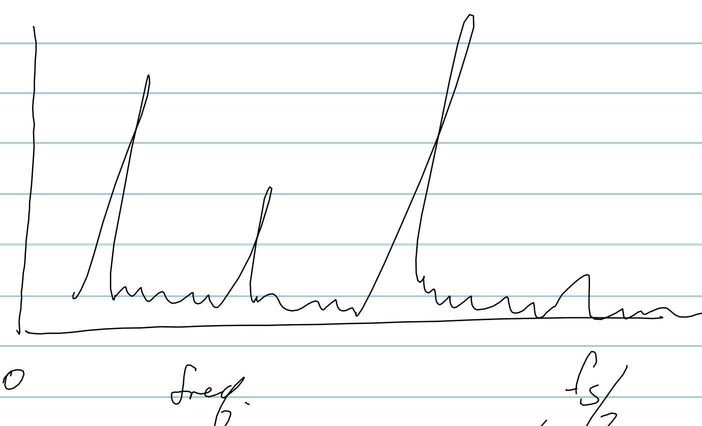
In practice, you may either to plot frequency response;

1. put in $A\sin(\omega t)$ and measure output where $T = \frac{\cancel{out}}{input}$

2. "Whack" and measure, then do a Fourier analysis

<u>FAST FOURIER TRANSFORM "FFT"</u>

"Windowing" → no discontinuity



$\Delta f = \frac{1}{T}$ → sample window

$f_s/2$ → half sampling rate

$$x(t) = 3\cos(\overset{K\omega_1}{5t}) + 7\cos(\underset{K\omega_1}{15t})$$

$\omega_1 = 5$ because it's the smallest integer that goes into $5t$ and $15t$

$A_1 = 3$

$A_3 = 7$

$$x(t) = 3 + 4\cos(6t) + 2\sin(7t)$$

$\omega_1 = 1$ (rad/s)

$\frac{A_0}{2} = 3$

$K=6$

$A_6 = 4$

$B_7 = 2$

$K=7$

Then you could plot if you want to

Integer multiples, mods, special constraints,

→ zero @ the ends

Homework due Friday

On Wed we'll get practice exam, then final on Friday

~~Wednesday June~~

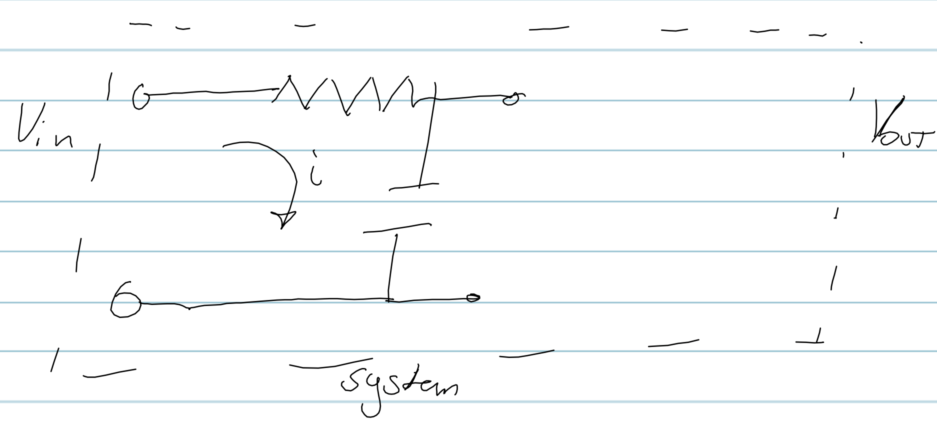
WEDNESDAY JUNE 5th, 2013

Homework #6 due Friday

Test Review Assignment Due Friday

FILTERS

$$T = \frac{\text{output}}{\text{Input}}$$



	Component	impedence
Resistor	R	R
Capacitor	C	$\frac{1}{Cj\omega}$
	L	$Lj\omega$

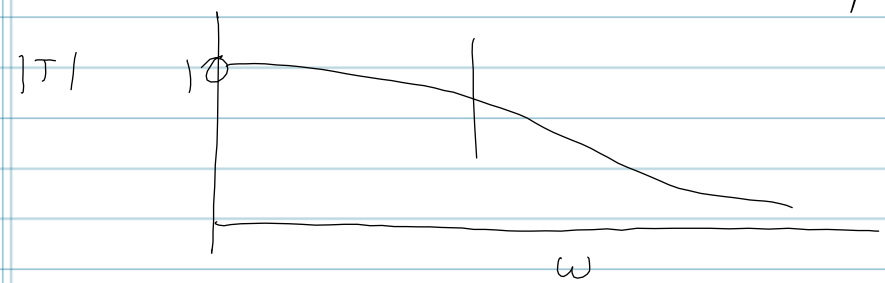
"KVL"

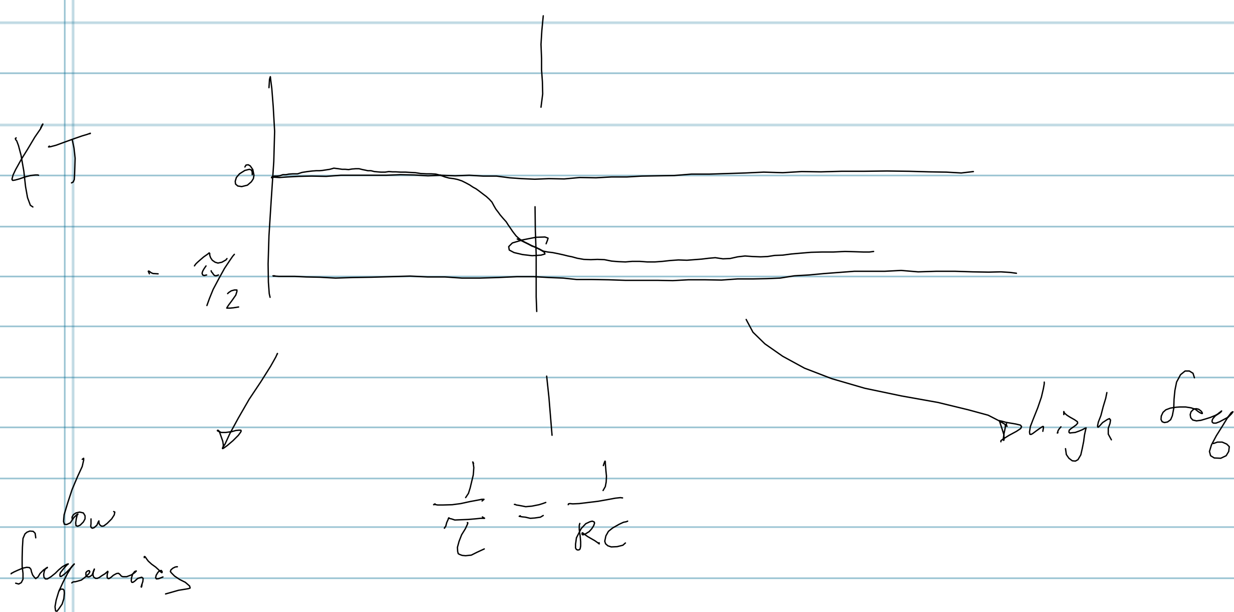
$$\left.\begin{aligned} V_{in} &= i\left(R + \frac{1}{Cj\omega}\right) \\ V_{out} &= i\left(\frac{1}{Cj\omega}\right) \end{aligned}\right\} T = \frac{V_{out}}{V_{in}} = \frac{i\left(\frac{1}{Cj\omega}\right)}{i\left(R + \frac{1}{Cj\omega}\right)} = \frac{1}{1 + RCj\omega}$$

1st order

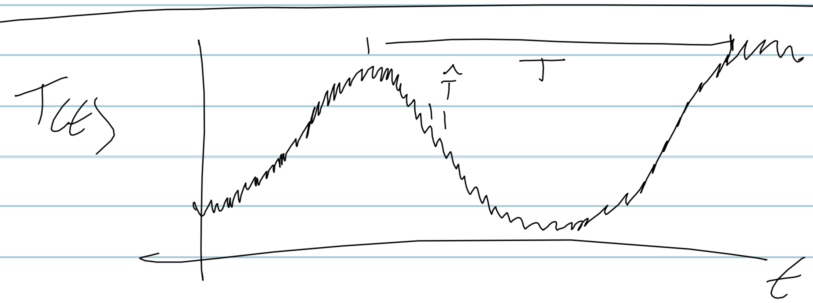
low freq.
↓ good

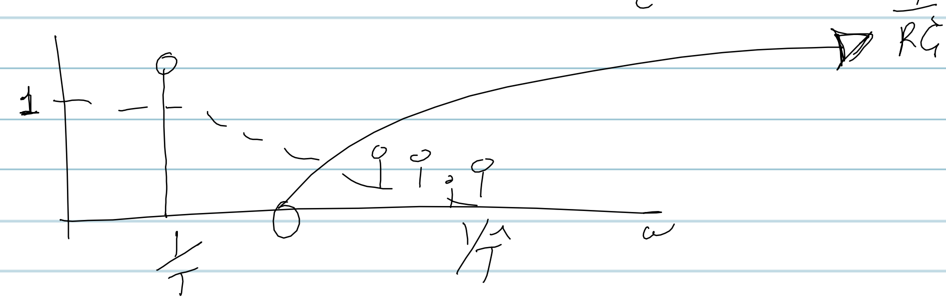
high freq.
↓ multiplied by ∠1.





<u>NOTE:</u> you can get rid of "hiss" by using a simple filter



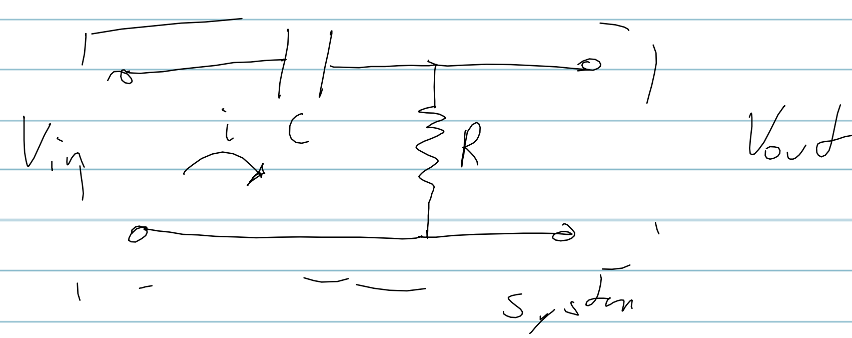


All physical systems behave like filters

→ Vowels vs. consonants, high v low frequency

ssss, → hiss from record player

HIGH-PASS FILTER

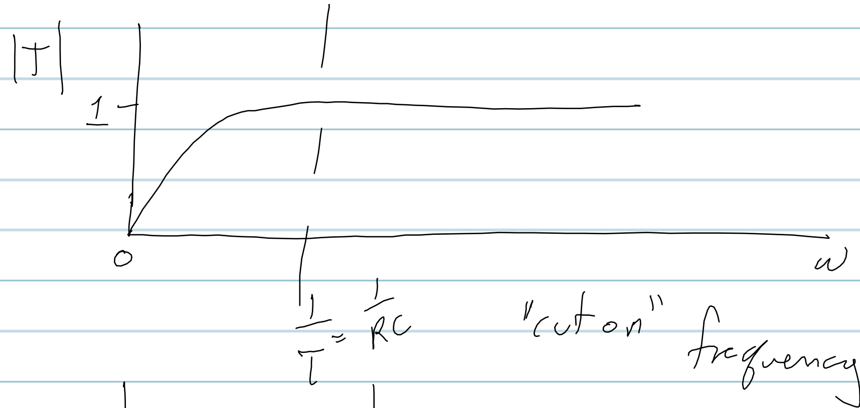


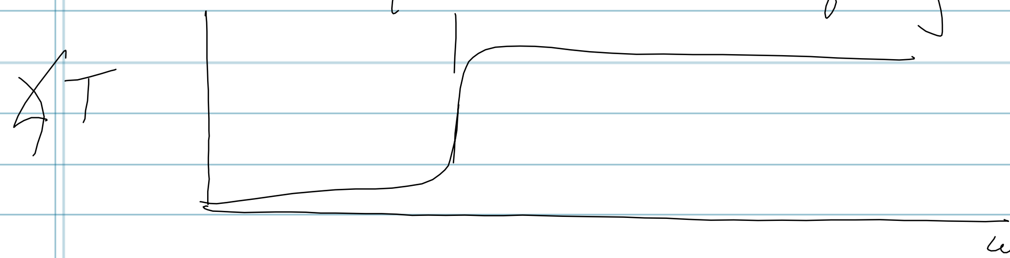
Now $V_{in} = i\left(\frac{1}{Cj\omega} + R\right)$

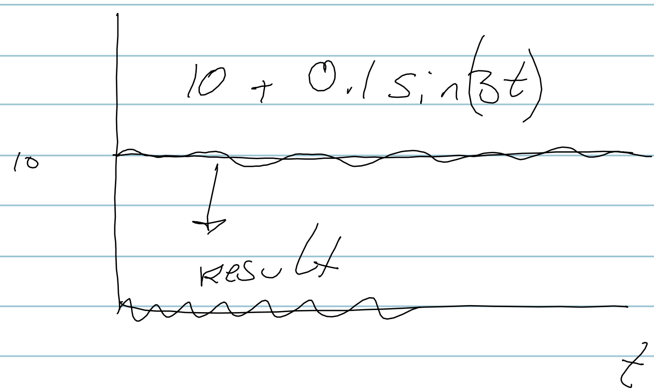
$V_{OUT} = iR$

$$\frac{V_{OUT}}{V_{in}} = \frac{R}{R+\frac{1}{Cj\omega}} = \frac{RCj\omega}{1+RCj\omega}$$

⇒

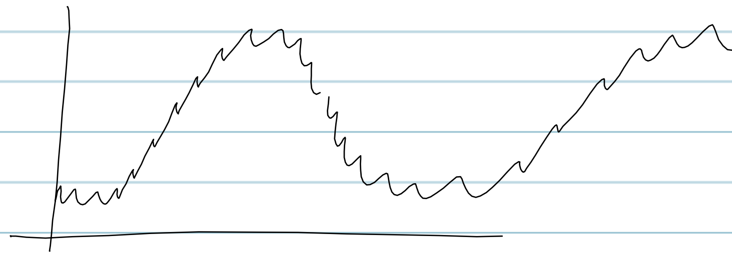




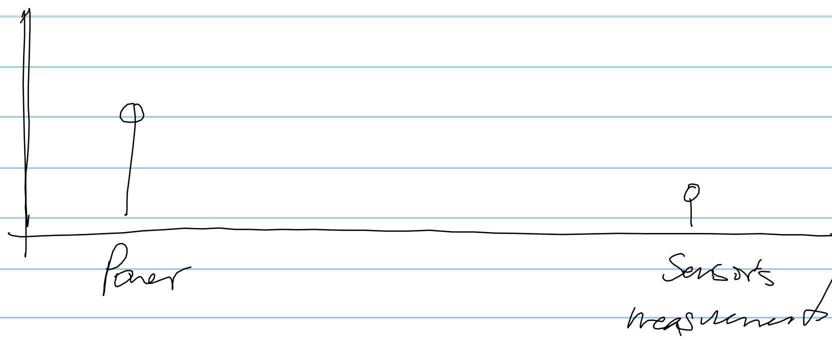


AC coupling → "high-pass" filter

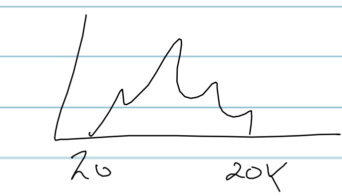
These can also be used in Amplifiers:

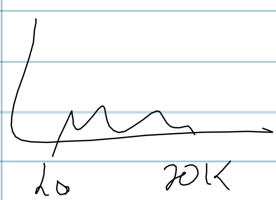


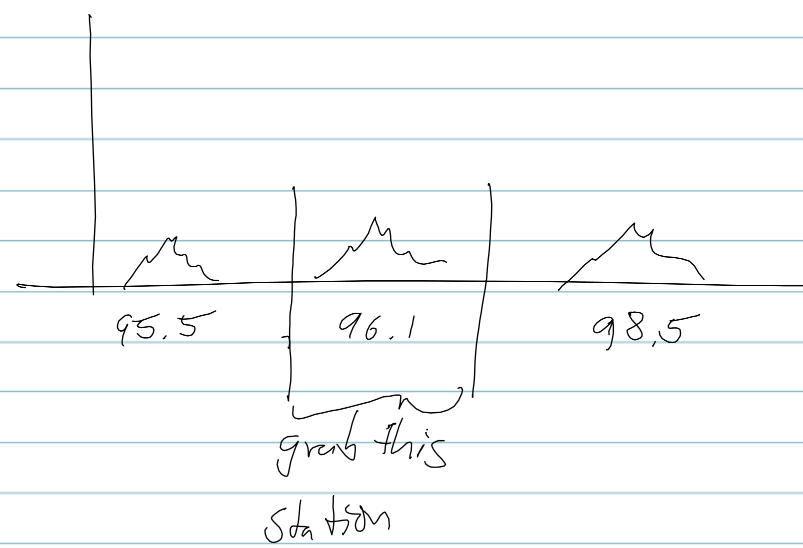
Modulation



↳ get rid of "Power" & leave sensor info

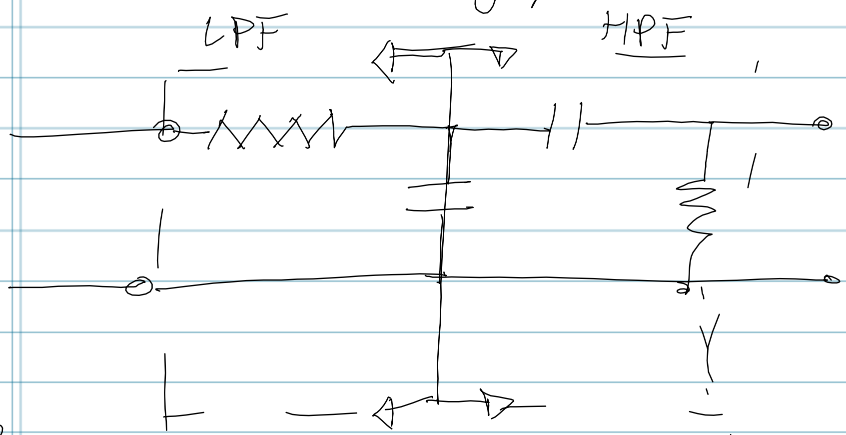






Band-pass filter →

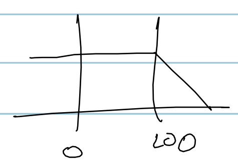
Put low-pass & high-pass filter in series

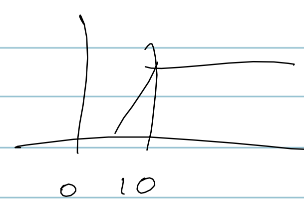


0 to ∞

10 to 100

0 to 100





↳ increase gain on band in that frequency range

↳ Channels on your stereo

NOTE: 4 Hz degraded spire in drivers

↳ "Better seat" (in 3 to 5 Hz range)

Friday June 7th, 2013

frf uses a sine wave, but via Fourier, we can convert any input into a sine wave

Static Calib → K. input → repeat
→ increasing & decreasing

Range
Input resolution

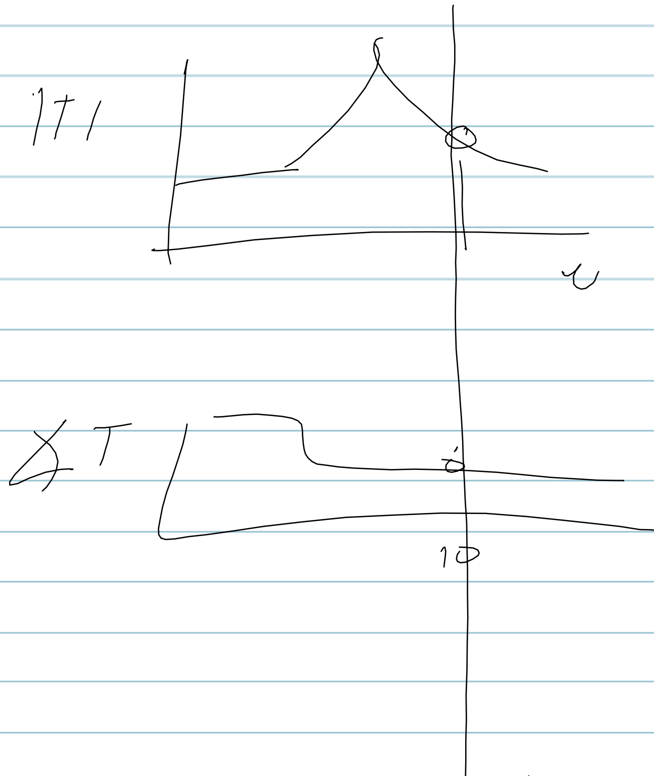
Statistics 1. conf. intvs.
2. Curve, fitting
3. Propagation

ADC Sampling 1, input range + pros
− cons
2, bits + pros
− cons
3. Sample rate → aliasing

Dynamic Models → Diff. eq. & fr function
req & req freq

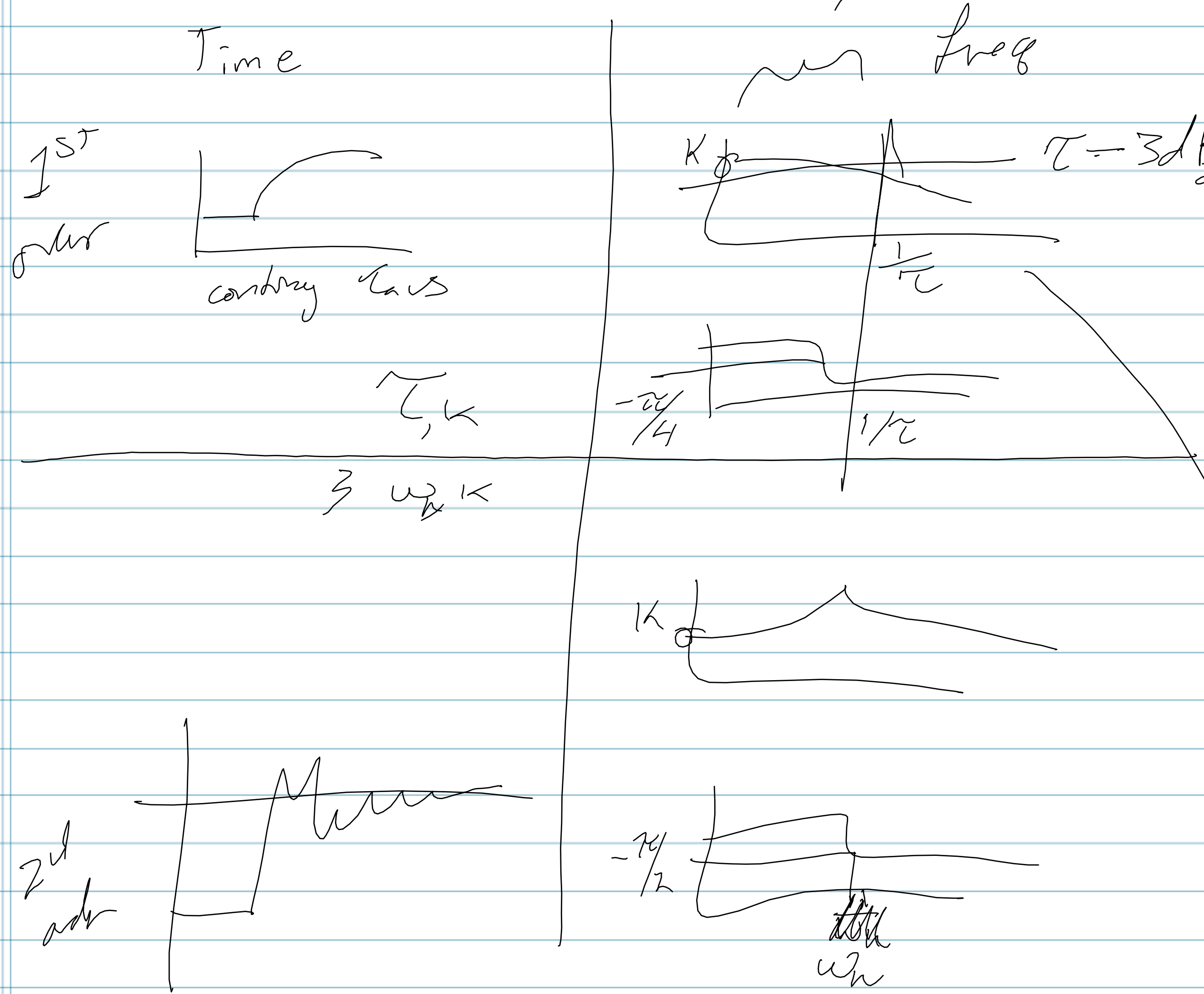
Magnitude & phase

$6 \sin 10t$ $\qquad$ $6|T|\sin(10t + \angle T)$



If you know your box, system, you know input & output & vice versa

Usable Range



τ = 3dB dropoff on Bode

↳ We can now determine the usable range

calibration relevence

Fourier

→ Knowing frequencies

↳ how we can use for any input