Bernoulli's Equation (Section 2.6)

The differential equation

$$\frac{dy}{dx} + P(x)y = f(x)y^n$$

where n is a real number is called **Bernoulli's equation**. (Note that when $n = 0$ or $n = 1$, the equation is linear.) For $y \neq 0$, we can divide through by y^n and write the equation can be written as follows.

$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = f(x)$$

If we let $u = y^{1-n}$, $n \neq 0$ and $n \neq 1$, then $\frac{du}{dx} = (1-n)y^{-n}\frac{dy}{dx}$. Making substitutions, we obtain the following linear equation that can be solved using the method of the integrating factor.

With this substitution, Bernoulli's equation becomes a linear equation.

$$y^{-n} \cdot \frac{1}{(1-n)y^{-n}}\frac{du}{dx} + P(x)u = f(x)$$

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)f(x)$$

Solving for u and using $u = y^{1-n}$ leads to a solution.

<u>Example 1:</u> Solve the Bernoulli equation $\frac{dy}{dx} = y(xy^3 - 1)$.