

13.3 \* 10

$$\vec{r}(t) = \langle 4+3t, 2-2t, 5+t \rangle; \quad 3 \leq t \leq 4$$

$$\vec{r}'(t) = \langle 3, -2, 1 \rangle$$

$$\Rightarrow \|\vec{r}'(t)\| = \sqrt{14}$$

$$\Rightarrow s = \int_3^4 \sqrt{14} dt = \sqrt{14} t \Big|_3^4 = \sqrt{14} \text{ length units} //$$

13.4 \* 10

$$\bar{r}(t) = \left\langle t, \frac{1}{2}t^2, \frac{1}{3}t^3 \right\rangle$$

$$\Rightarrow \bar{r}'(t) = \langle 1, t, t^2 \rangle \Rightarrow \|\bar{r}'(t)\| = \sqrt{1 + t^2 + t^4}$$

$$\Rightarrow \bar{T}(t) = \frac{\langle 1, t, t^2 \rangle}{\sqrt{1 + t^2 + t^4}} = \langle 1, t, t^2 \rangle (1 + t^2 + t^4)^{-\frac{1}{2}}$$

$$\bar{T}'(t) = \frac{\langle 0, 1, 2t \rangle}{\sqrt{1 + t^2 + t^4}} - \frac{1}{2} \frac{\langle 1, t, t^2 \rangle \cdot (2t + 4t^3)}{(1 + t^2 + t^4)^{\frac{3}{2}}}$$

$$\Rightarrow \bar{T}'(0) = \langle 0, 1, 0 \rangle$$

$$\Rightarrow \|\bar{T}'(0)\| = 1$$

$$\Rightarrow \bar{N}(0) = \langle 0, 1, 0 \rangle //$$

$$\bar{T}(0) = \langle 1, 0, 0 \rangle //$$

13.5 \* 48

$$y = e^x \rightarrow \frac{dy}{dx} = e^x \text{ and } \frac{d^2y}{dx^2} = e^x.$$

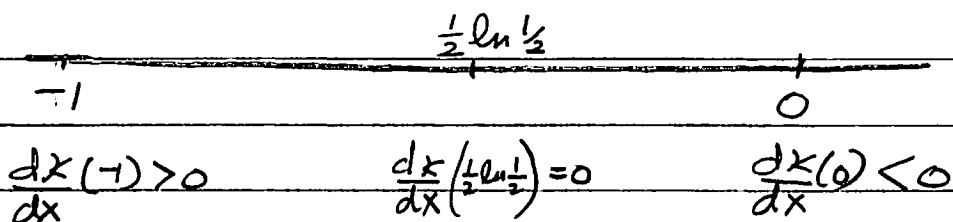
$$\Rightarrow x(x) = \frac{e^x}{(1+e^{2x})^{3/2}} \quad (\text{See prob. 19.})$$

$$\frac{dx}{dx} = \frac{e^x}{(1+e^{2x})^{3/2}} - \frac{3}{2} \frac{e^x \cdot 2e^{2x}}{(1+e^{2x})^{5/2}}$$

$$= \frac{e^x [1+e^{2x} - 3e^{2x}]}{(1+e^{2x})^{5/2}} = \frac{e^x [1-2e^{2x}]}{(1+e^{2x})^{5/2}}$$

$$\frac{dx}{dx} = 0 \Rightarrow e^{2x} = \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2} \ln\left(\frac{1}{2}\right)$$



$\therefore$  By 1<sup>st</sup> derivative test, curvature of  $y = e^x$  is maximum at  $x = \frac{1}{2} \ln \frac{1}{2}$ .

13.6 \*49

$$\vec{v} = \langle 3, 0, -4 \rangle, \quad \vec{a} = \langle 1, -1, 2 \rangle$$

$$a_T = \frac{\vec{v} \cdot \vec{a}}{\|\vec{v}\|} = \frac{3-8}{5} = -1 //$$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -4 \\ 1 & -1 & 2 \end{vmatrix} = \langle -4, -10, -3 \rangle$$

$$\Rightarrow \|\vec{v} \times \vec{a}\| = \sqrt{125} = 5\sqrt{5}$$

$$\therefore a_N = \frac{5\sqrt{5}}{5} = \sqrt{5} //$$

$$\vec{T} = \frac{\vec{v}}{\|\vec{v}\|} = \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle //$$

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

$$\Rightarrow \langle 1, -1, 2 \rangle = -1 \left\langle \frac{3}{5}, 0, -\frac{4}{5} \right\rangle + \sqrt{5} \vec{N}$$

$$\Rightarrow \sqrt{5} \vec{N} = \left\langle \frac{6}{5}, -1, \frac{6}{5} \right\rangle$$

$$\Rightarrow \vec{N} = \left\langle \frac{6}{5\sqrt{5}}, -\frac{1}{\sqrt{5}}, \frac{6}{5\sqrt{5}} \right\rangle //$$