13.3 #10

$\bar{r}(t) = \langle 4+3t, 2-2t, 5+t \rangle$; $3 \leq t \leq 4$

$\bar{r}'(t) = \langle 3, -2, 1 \rangle$

$\Rightarrow \|\bar{r}'(t)\| = \sqrt{14}$

$\Rightarrow s = \int_3^4 \sqrt{14}\,dt = \sqrt{14}\,t \Big|_3^4 = \sqrt{14}$ length units

13.4 #10

$\bar{r}(t) = \langle t, \frac{1}{2}t^2, \frac{1}{3}t^3 \rangle$

$\Rightarrow \bar{r}'(t) = \langle 1, t, t^2 \rangle \Longrightarrow \|\bar{r}'(t)\| = \sqrt{1+t^2+t^4}$

$\Rightarrow \bar{T}(t) = \dfrac{\langle 1, t, t^2 \rangle}{\sqrt{1+t^2+t^4}} = \langle 1, t, t^2 \rangle (1+t^2+t^4)^{-\frac{1}{2}}$

$\bar{T}'(t) = \dfrac{\langle 0, 1, 2t \rangle}{\sqrt{1+t^2+t^4}} - \dfrac{1}{2} \dfrac{\langle 1, t, t^2 \rangle \cdot (2t+4t^3)}{(1+t^2+t^4)^{3/2}}$

$\Rightarrow \bar{T}'(0) = \langle 0, 1, 0 \rangle$

$\Rightarrow \|\bar{T}'(0)\| = 1$

$\Rightarrow \bar{N}(0) = \langle 0, 1, 0 \rangle$ //

$\bar{T}(0) = \langle 1, 0, 0 \rangle$ //

13.5 #48

$$y = e^x \longrightarrow \frac{dy}{dx} = e^x \text{ and } \frac{d^2y}{dx^2} = e^x.$$

$$\Longrightarrow \kappa(x) = \frac{e^x}{(1+e^{2x})^{3/2}} \quad \text{(See prob. 19.)}$$

$$\frac{d\kappa}{dx} = \frac{e^x}{(1+e^{2x})^{3/2}} - \frac{3}{2}\frac{e^x \cdot 2e^{2x}}{(1+e^{2x})^{5/2}}$$

$$= \frac{e^x\left[1+e^{2x}-3e^{2x}\right]}{(1+e^{2x})^{5/2}} = \frac{e^x\left[1-2e^{2x}\right]}{(1+e^{2x})^{5/2}}$$

$$\frac{d\kappa}{dx} = 0 \Longrightarrow e^{2x} = \frac{1}{2}$$

$$\Longrightarrow x = \frac{1}{2}\ln\left(\frac{1}{2}\right)$$

$\quad -1 \qquad\qquad \frac{1}{2}\ln\frac{1}{2} \qquad\qquad 0$

$\frac{d\kappa}{dx}(-1) > 0 \qquad \frac{d\kappa}{dx}\left(\frac{1}{2}\ln\frac{1}{2}\right) = 0 \qquad \frac{d\kappa}{dx}(0) < 0$

$\therefore$ By 1^{st} derivative test, curvature of $y = e^x$ is maximum at $x = \frac{1}{2}\ln\frac{1}{2}$. //

13.6 #42

$\bar{v} = \langle 3, 0, -4 \rangle, \quad \bar{a} = \langle 1, -1, 2 \rangle$

$$a_T = \frac{\bar{v} \cdot \bar{a}}{\|\bar{v}\|} = \frac{3-8}{5} = -1 \;//$$

$$\bar{v} \times \bar{a} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 0 & -4 \\ 1 & -1 & 2 \end{vmatrix} = \langle -4, -10, -3 \rangle$$

$$\Rightarrow \|\bar{v} \times \bar{a}\| = \sqrt{125} = 5\sqrt{5}$$

$$\therefore a_N = \frac{5\sqrt{5}}{5} = \sqrt{5} \;//$$

$$\bar{T} = \frac{\bar{v}}{\|\bar{v}\|} = \left\langle \frac{3}{5}, 0, \frac{-4}{5} \right\rangle \;//$$

$$\bar{a} = a_T \bar{T} + a_N \bar{N}$$

$$\Rightarrow \langle 1, -1, 2 \rangle = -1 \left\langle \frac{3}{5}, 0, \frac{-4}{5} \right\rangle + \sqrt{5}\, \bar{N}$$

$$\Rightarrow \sqrt{5}\, \bar{N} = \left\langle \frac{8}{5}, -1, \frac{6}{5} \right\rangle$$

$$\Rightarrow \bar{N} = \left\langle \frac{8}{5\sqrt{5}}, \frac{-1}{\sqrt{5}}, \frac{6}{5\sqrt{5}} \right\rangle \;//$$