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$$T(x,y) = \frac{xy}{1+x^2+y^2} = xy\,(1+x^2+y^2)^{-1}$$

$$\Rightarrow \bar{\nabla} T(x,y) = \left\langle \frac{y}{1+x^2+y^2} - \frac{2x^2y}{(1+x^2+y^2)^2},\ \frac{x}{1+x^2+y^2} - \frac{2xy^2}{(1+x^2+y^2)^2} \right\rangle$$

$$\Rightarrow \bar{\nabla} T(1,1) = \left\langle \frac{1}{3} - \frac{2}{9},\ \frac{1}{3} - \frac{2}{9} \right\rangle$$

$$= \left\langle \frac{1}{9}, \frac{1}{9} \right\rangle$$

$$\bar{a} = \langle 2,-1\rangle \Longrightarrow \bar{u} = \frac{\bar{a}}{\|\bar{a}\|} = \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$$

(a) $$D_{\bar{u}} T(1,1) = \bar{\nabla} T(1,1) \cdot \bar{u}$$

$$= \frac{2}{9\sqrt{5}} - \frac{1}{9\sqrt{5}} = \frac{1}{9\sqrt{5}} \,//$$

(b) $$\|\bar{\nabla} T(1,1)\| = \sqrt{\frac{1}{81} + \frac{1}{81}} = \frac{\sqrt{2}}{9}$$

$$\Rightarrow \frac{\bar{\nabla} T(1,1)}{\|\bar{\nabla} T(1,1)\|} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$\therefore$ The ant must walk in the direction of $\left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$ at $(1,1)$. //

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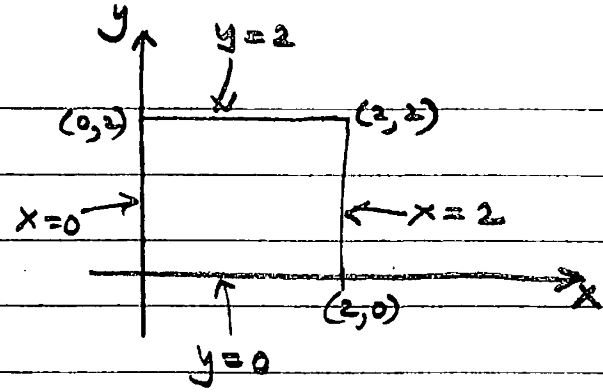
$f(x,y) = x^2 - 3y^2 - 2x + 6y$

$f_x = 2x - 2$

$f_y = -6y + 6$



$f_x = 0$ & $f_y = 0 \Longrightarrow (1,1)$ is an interior c.p.

$f(x,0) = x^2 - 2x$

$\frac{df}{dx} = 2x - 2 = 0 \Longrightarrow (1,0)$ is a c.p. on $y = 0$.

$f(0,y) = -3y^2 + 6y$

$\frac{df}{dy} = -6y + 6 = 0 \Longrightarrow (0,1)$ is a c.p on $x = 0$.

$f(x,2) = x^2 - 2x$

$\frac{df}{dx} = 0 \Longrightarrow (1,2)$ is a c.p. on $y = 2$.

$f(2,y) = -3y^2 + 6y$

$\frac{df}{dy} = 0 \Longrightarrow (2,1)$ is a c.p on $x = 2$.

(x,y)	$(1,1)$	$(1,0)$	$(0,1)$	$(1,2)$	$(2,1)$	$(0,0)$	$(2,0)$	$(2,2)$	$(0,2)$
$f(x,y)$	2	-1	3	-1	3	0	0	0	0

$\Longrightarrow$ f has an abs. max at $(0,1)$ and $(2,1)$ and the abs max. is 3. //

f has an abs. min. at $(1,0)$ and $(1,2)$ and the abs. min is -1 //

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8

Let $g(x,y) = x^2+3y^2-16$

$\Rightarrow \bar{\nabla} g(x,y) = \langle 2x, 6y \rangle$

$\bar{\nabla} f(x,y) = \langle 1, -3 \rangle$

$\bar{\nabla} f(x,y) = \lambda \bar{\nabla} g(x,y) \Longrightarrow \langle 1, -3 \rangle = \lambda \langle 2x, 6y \rangle$

$\Longrightarrow \quad 1 = 2x\lambda$ —(1)

$-3 = 6y\lambda$ —(2)

$x^2+3y^2-16=0$ —(3)

$\lambda \neq 0 \Longrightarrow -\frac{1}{3} = \frac{x}{3y}$ from (1) & (2).

$\Longrightarrow \quad x = -y$

Then $y^2+3y^2-16=0$ from (3).

$\therefore y = \pm 2$ and $x = \mp 2$

(x,y)	$(2,-2)$	$(-2,2)$
$f(x,y)$	7	-9

$\therefore$ f has a const. rel. max. at $(2,-2)$ and it is 7 //

f has a const. rel. min. at $(-2,2)$ and it is -9. //