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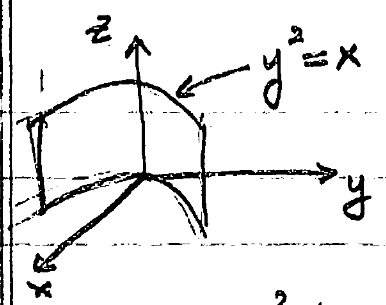
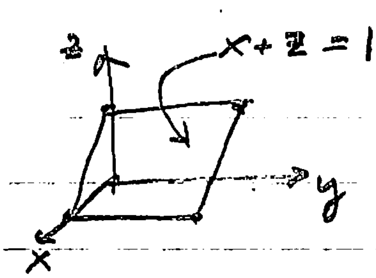
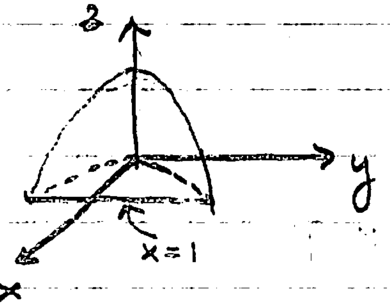
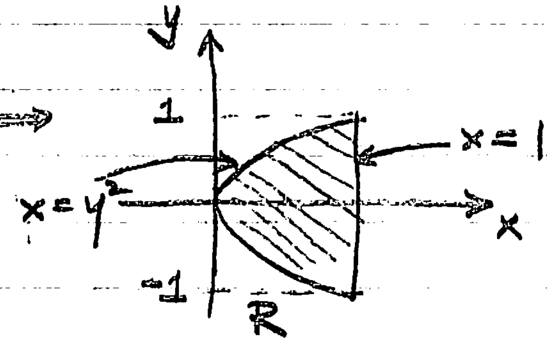




$\Longrightarrow$



$\Longrightarrow$



$$\Rightarrow \iint_R f(x,y)\,dA = \int_{-1}^{1}\int_{y^2}^{1} 1-x\,dx\,dy$$

$$= \int_{-1}^{1} x-\frac{x^2}{2}\Big|_{y^2}^{1}\,dy$$

$$= \int_{-1}^{1} \frac{1}{2} - y^2 + \frac{y^4}{2}\,dy$$

$$= \frac{1}{2}y - \frac{1}{3}y^3 + \frac{1}{10}y^5\Big|_{-1}^{1}$$

$$= \left(\frac{1}{2}-\frac{1}{3}+\frac{1}{10}\right) - \left(-\frac{1}{2}+\frac{1}{3}-\frac{1}{10}\right)$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$= \frac{8}{15} \text{ vol units} \;//$$

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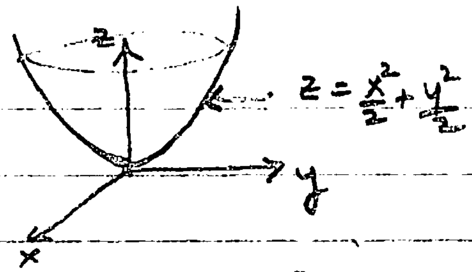
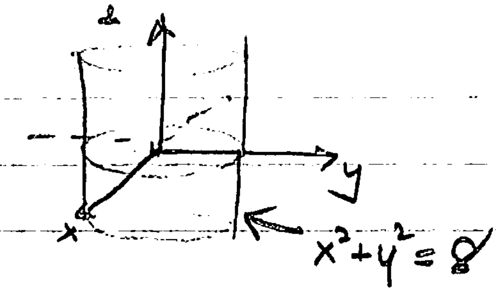
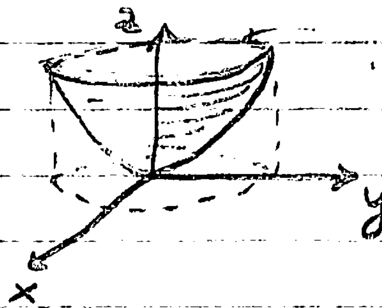
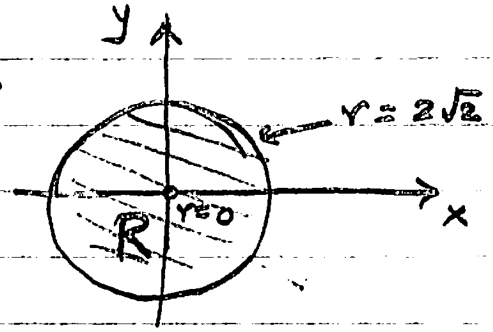




$\Rightarrow$



$\Rightarrow$



$z_x = x, \quad z_y = y$

$\Rightarrow \sqrt{z_x^2 + z_y^2 + 1} = \sqrt{x^2+y^2+1}$

$\therefore$ Surface area $= \iint_R \sqrt{x^2+y^2+1}\, dA$

$$= \int_0^{2\pi}\int_0^{2\sqrt{2}} \sqrt{r^2+1}\; r\,dr\,d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} (r^2+1)^{3/2}\Big|_0^{2\sqrt{2}}\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} 9^{3/2}\, d\theta$$

$$= 9 \cdot 2\pi$$

$$= 18\pi \text{ square units}$$

$u = r^2+1$
$du = 2r\,dr$
$\Rightarrow \int \sqrt{r^2+1}\; r\,dr$
$= \frac{1}{2}\int u^{1/2}\,du$
$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$
$= \frac{1}{3}(r^2+1)^{3/2} + C$