MTH 202: Quiz 2 **Name:** key

Time: 30 minutes

Use of notes, note cards, cellphones or PDAs are not allowed during the quiz.

(1) Find the vector equation of the tangent line to $r(t) =< \cos t, 2\sin t, 2t >$ at the point where $t = \frac{\pi}{6}$.

Show all the reasoning, in a step by step manner, which leads to the logical conclusion.

$\bar{r}'(t) = \langle -\sin t, 2\cos t, 2 \rangle$

$\Rightarrow \bar{r}'(\frac{\pi}{6}) = \langle -\frac{1}{2}, \sqrt{3}, 2 \rangle$

$\bar{r}(\frac{\pi}{6}) = \langle \frac{\sqrt{3}}{2}, 1, \frac{\pi}{3} \rangle$

Therefore, the vector equation of the tangent line is:

$\bar{r}(t) = \langle \frac{\sqrt{3}}{2}, 1, \frac{\pi}{3} \rangle + t\langle -\frac{1}{2}, \sqrt{3}, 2 \rangle$ for some t

or

$\bar{r}(t) = \langle \frac{\sqrt{3}}{2} - \frac{1}{2}t, 1 + \sqrt{3}t, \frac{\pi}{3} + 2t \rangle$ for some t. //

(2) Find the vector equation of $\bar{r}(t) =< 3t, 4 - t, t + 1 >$ using arc length s as a parameter. Use the point on the curve where $t = 0$ as the reference point.

Show all your reasoning, in a step by step manner, which leads to the logical conclusion.

$\bar{r}'(t) = \langle 3, -1, 1 \rangle$

$\Rightarrow \|\bar{r}'(t)\| = \sqrt{11}$

$\Rightarrow s = \int_0^t \sqrt{11}\, dt$

$\Rightarrow s = \sqrt{11}\, t$

$\therefore t = \frac{s}{\sqrt{11}}$

$\Rightarrow \bar{r}(s) = \langle \frac{3}{\sqrt{11}}s, 4 - \frac{1}{\sqrt{11}}s, \frac{1}{\sqrt{11}}s + 1 \rangle$ //

(3) Find the curvature of $\bar{r}(t) = < t, t^2, t^3 >$ at $t = 1$.
Show all your reasoning, in a step by step manner, which leads to the logical conclusion.

$$\bar{r}'(t) = <1, 2t, 3t^2> \implies \bar{r}'(1) = <1, 2, 3>$$
$$\bar{r}''(t) = <0, 2, 6t> \implies \bar{r}''(1) = <0, 2, 6>$$
$$\implies \bar{r}'(t) \times \bar{r}''(t) = <6, -6, 2>$$
$$\implies \|\bar{r}'(t) \times \bar{r}''(t)\| = \sqrt{76} = 2\sqrt{19}$$
$$\|\bar{r}'(t)\| = \sqrt{14} \implies \|\bar{r}'(t)\|^3 = 14\sqrt{14}.$$
$$\implies \kappa(1) = \frac{\sqrt{19}}{7\sqrt{14}} //$$

(4) A particle moves through 3-space in such a way that it's velocity $\bar{v}(t) = < 2, -4t^3, 6\sqrt{t} >$. If the particle was initially (i.e. when $t = 0$) at the point $(1, 5, 3)$ then find the position (coordinates of the point) of the particle when $t = 1$.
Show all your reasoning, in a step by step manner, which leads to the logical conclusion.

Let $\bar{r}(t)$ be the position function of the particle.

Then $\bar{r}(t) = \int <2, -4t^3, 6t^{1/2}> \, dt$

$$= <2t, -t^4, 4t^{3/2}> + \bar{C}$$

Since, $\bar{r}(0) = <1, 5, 7>$, $\bar{C} = <1, 5, 3>$.

$$\implies \bar{r}(t) = <2t+1, 5-t^4, 4t^{3/2}+3>$$
$$\implies \bar{r}(1) = <3, 4, 7>$$

$\therefore$ The particle will be at $(3, 4, 7)$ when $t = 1$. //