MTH 202: Quiz 3 **Name:** Key
Time: 30 minutes

Use of notes, note cards, cellphones, PDAs or symbolic calculators are not allowed during the quiz.

(1) Find the equation of the tangent plane and the parametric equations of the normal line to $z = \ln\sqrt{x^2+y^2}$ at the point $(-1,0,0)$. Show all the work as you have learned in the class.

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x = \frac{x}{x^2+y^2} \text{ \& by symm. } \frac{\partial z}{\partial y} = \frac{y}{x^2+y^2}.$$

$$\frac{\partial z}{\partial x}(-1,0,0) = -1, \quad \frac{\partial z}{\partial y}(-1,0,0) = 0.$$

∴ The equation of the tangent line to z at $(-1,0,0)$ is:

$-1(x+1) - z = 0 \implies x+z+1 = 0$ //

Parametric equations of the tangent line are:

$$\left.\begin{aligned} x(t) &= -1-t \\ y(t) &= 0 \\ z(t) &= -t \end{aligned}\right\} \text{ for some parameter } t.$$

(2) Use Lagrange Multiplier method to find the maximum and minimum values of $f(x,y) = 4x^3 + y^2$ subject to the constraint $2x^2 + y^2 = 1$. Show all the work as you have learned in the class.

Let $g(x,y) = 2x^2+y^2-1$.

Then $\overline{\nabla} g(x,y) = \langle 4x, 2y\rangle$.

$\overline{\nabla} f(x,y) = \langle 12x^2, 2y\rangle$

$\overline{\nabla} f = \lambda \overline{\nabla} g \implies$

$3x^2 = \lambda x$ —(1)

$y = \lambda y$ —(2)

and the constraint

$2x^2+y^2 = 1$ —(3)

Suppose $y \neq 0$.

Then $\lambda = 1$. Then by (1)

$x = 0$ or $x = \frac{1}{3}$

$x = 0 \implies y = \pm 1$

$x = \frac{1}{3} \implies y = \pm\frac{\sqrt{7}}{3}$

Suppose $y = 0$.

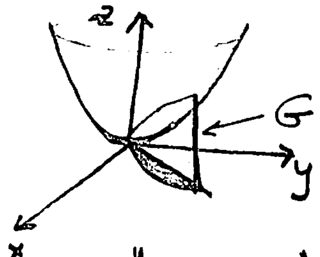
Then $x = \pm\frac{1}{\sqrt{2}}$.

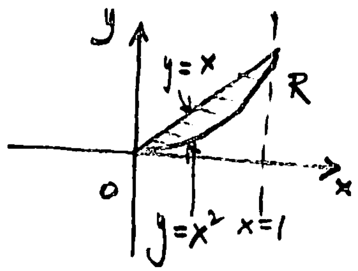
(x,y)	$(0,1)$	$(0,-1)$	$(\frac{1}{3},\frac{\sqrt{7}}{3})$	$(\frac{1}{3},-\frac{\sqrt{7}}{3})$	$(\frac{1}{\sqrt{2}},0)$	$(-\frac{1}{\sqrt{2}},0)$
$f(x,y)$	1	1	$\frac{25}{27}$	$\frac{25}{27}$	$\sqrt{2}$	$\sqrt{2}$

∴ f has a const. rel. max. at $(\frac{1}{\sqrt{2}},0)$ or at $(-\frac{1}{\sqrt{2}},0)$ and the const. rel. max. is $\sqrt{2}$. //

f has a const. rel. min. at $(\frac{1}{3},\frac{\sqrt{7}}{3})$ or at $(\frac{1}{3},-\frac{\sqrt{7}}{3})$ and the const. rel. min. is $\frac{25}{27}$. //

(3) Find the volume of the solid in the first octant bounded above by the paraboloid $z = x^2 + 3y^2$, below the plane $z = 0$ and laterally by the planes $y = x^2$ and $y = x$. Show all the work including the sketch of the solid and the region R as you have learned in the class.





$$\iint_R x^2+3y^2\,dA = \int_0^1\int_{x^2}^{x} x^2+3y^2\,dy\,dx$$

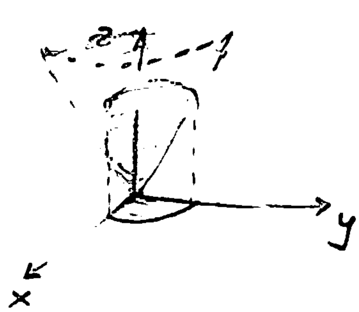
$$= \int_0^1 x^2y + y^3\Big|_{x^2}^{x}\,dx$$

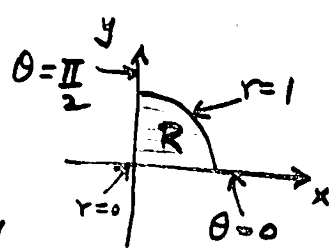
$$= \int_0^1 2x^3 - (x^4 + x^6)\,dx$$

$$= \frac{x^4}{2} - \frac{x^5}{5} - \frac{x^7}{7}\Big|_0^1 = \frac{1}{2} - \frac{1}{5} - \frac{1}{7}$$

$$= \frac{35-14-10}{70} = \frac{11}{70} \text{ vol. units}$$

(4) Find the area of the portion of the surface $z^2 = 4x^2 + 4y^2$ that is above the region in the first quadrant inside the curve $x^2 + y^2 = 1$. Sketch the surface and the region R. Show all the work including the sketch of the surface and the region R as you have learned in the class.





$$2z\frac{\partial z}{\partial x} = 8x \Rightarrow \frac{\partial z}{\partial x} = \frac{4x}{z}$$

$$2z\frac{\partial z}{\partial y} = 8y \Rightarrow \frac{\partial z}{\partial y} = \frac{4y}{z}$$

$$\Rightarrow \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} = \sqrt{\frac{16x^2+16y^2}{z^2}+1}$$

$$= \sqrt{\frac{4z^2}{z^2}+1} = \sqrt{5}$$

$$\therefore \text{Surface area} = \int_0^{\pi/2}\int_0^1 \sqrt{5}\,r\,dr\,d\theta$$

$$= \frac{\sqrt{5}}{2}\cdot\frac{\pi}{2}$$

$$= \frac{\sqrt{5}\pi}{4} \text{ area units}$$