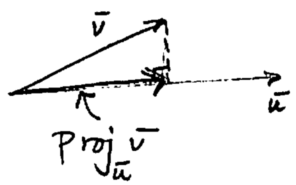
MTH 202: Quiz 1 **Name:** key
Time: 30 minutes

Use of notes, note cards, cellphones or PDAs are not allowed during the quiz.

(1) Find the projection of $\overline{v} = < 1, 0, 2 >$ in the direction of $\overline{u} = < 2, -1, 3 >$
Show all the reasoning, in a step by step manner, which leads to the logical conclusion.



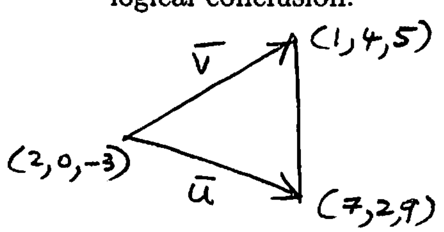
$$\text{Proj}_{\bar{u}}\bar{v} = \frac{\bar{v}\cdot\bar{u}}{\|\bar{u}\|^2}\,\bar{u}$$

$$= \frac{8}{14}\langle 2, -1, 3\rangle$$

$$= \frac{4}{7}\langle 2, -1, 3\rangle$$

$\bar{u}\cdot\bar{v} = 2 + 0 + 6$
$= 8$
$\|\bar{u}\| = \sqrt{4+1+9}$
$= \sqrt{14}$

(2) Find the area of the triangle with vertices $(2, 0, -3)$, $(1, 4, 5)$ and $(7, 2, 9)$ <u>using vectors</u>. Show all your reasoning, in a step by step manner, which leads to the logical conclusion.



Let $\bar{u} = \langle 5, 2, 12\rangle$ and
$\bar{v} = \langle -1, 4, 8\rangle$. (See figure.)

$\Rightarrow \bar{u}\times\bar{v} = \langle -32, -52, 22\rangle$

$\Rightarrow \bar{u}\times\bar{v} = 2\langle -16, 26, 11\rangle$

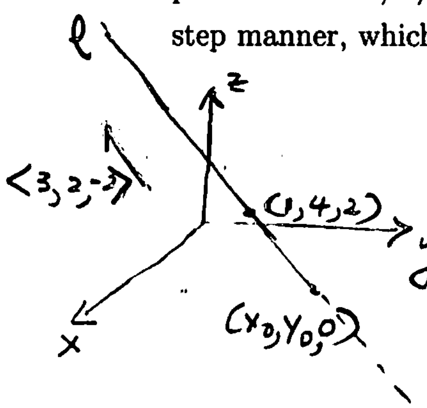
$\Rightarrow \|\bar{u}\times\bar{v}\| = 2\sqrt{16^2+26^2+11^2}$

$= 2\sqrt{1053}$

$\therefore$ Area of the triangle is $\sqrt{1053}$ s.u.

or $9\sqrt{13}$ s.u.

(3) Find the point where the line which passes through the point $(1, 4, 2)$ and is parallel to $< 3, 2, -2 >$ pierces the $xy-$plane. Show all your reasoning, in a step by step manner, which leads to the logical conclusion.



Let l be the line.
$\Rightarrow$ The equation of l is:

$$\langle x, y, z\rangle = \langle 1, 4, 2\rangle + t\langle 3, 2, -2\rangle,$$

for some parameter t.

Let $(x_0, y_0, 0)$ be the pt of intersection of l with the xy-plane.

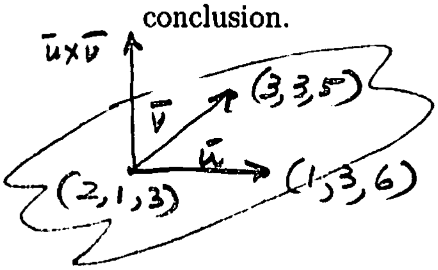
Then for some t_0, $\langle x_0, y_0, 0\rangle = \langle 1, 4, 2\rangle + t_0\langle 3, 2, -2\rangle$.

Comparing z-compt: $0 = 2 - 2t_0 \Longrightarrow t_0 = 1$

$\therefore$ $x_0 = 4$ and $y_0 = 6$.

$\Longrightarrow$ $(x_0, y_0, 0) \equiv (4, 6, 0)$ //

(4) Find the equation of the plane containing the points $(2, 1, 3)$, $(3, 3, 5)$, and $(1, 3, 6)$. Show all your reasoning, in a step by step manner, which leads to the logical conclusion.



Let $\bar{u} = \langle -1, 2, 3\rangle$ and
$\bar{v} = \langle 1, 2, 2\rangle$. (See figure.)

$\Longrightarrow$ $\bar{u} \times \bar{v} = \langle -2, 5, -4\rangle$

$\therefore$ The equation of the plane is:

$$-2(x-2) + 5(y-1) - 4(z-3) = 0$$

or $\quad 2x - 5y + 4z - 11 = 0$ //